CHAPTER 10

WAVE OPTICS

10.1 NATURE OF LIGHT : INTRODUCTORY CONCEPTS

1. *Briefly discuss the various theories about the nature of light.*

Nature of light. Various theories about the nature of light have been proposed from time to time. Some of the main theories are as follows :

1. ***Corpuscular theory of light.*** *Newton*, the great among the greatests, proposed in 1675 A.D. that light consists of tiny particles called *corpuscles* which are shot out at high speed by a luminous object. This theory could explain the reflection, refraction and rectilinear propagation of light.

2. ***Wave theory of light.*** In 1678, Dutch scientist *Christian Huygens*, suggested that light travels in the form of longitudinal waves just as sound propagates through air. He proposed that light waves propagate through an all-pervading hypothetical medium, called *luminiferous ether*. Later on, the existence of such a medium was discarded due to its contradictory properties. *Fresnel* and *Young* showed that light propagates as a transverse wave. This successfully explained the reflection, refraction as well as interference, diffraction and polarisation of light waves.

3. ***Electromagnetic nature of light waves.*** In 1873, *Maxwell* suggested that light propagates as electric and magnetic field oscillations. These are called electromagnetic waves which require no medium for their propagation. Also, these waves are transverse in nature.

4. ***Planck's quantum theory of light.*** According to *Max Planck*, light travels in the form of small packets of energy called photons. In 1905, *Albert Einstein* used this theory to explain photoelectric effect (emission of electrons from a metal surface when light falls on it).

So we see that in phenomena like interference, diffraction and polarisation, light behaves as a wave while in photoelectric effect, it behaves as a particle. *de Broglie* suggested that *light has a dual nature, i.e.*, it can behave as particles as well as waves.

10.2 WAVEFRONTS AND RAYS

2. *Distinguish between a wavefront and a ray of light. What are spherical, cylindrical and plane wavefronts ? Give their examples, sketch wavefronts corresponding to parallel, converging and diverging rays of light.*

Wavefronts. Suppose a stone is thrown on the surface of still water. Circular patterns of alternate crests and troughs begin to spread out from the point of impact. Clearly, all the particles lying on a crest are in the position of their maximum upward displacement and hence in the same phase. Similarly, all particles lying on a trough are in the position of their maximum downward displacement and, therefore, in the same phase. The locus of all such points oscillating in the same phase is called a ***wavefront***. Thus every crest or a trough is a wavefront.

A wavefront is defined as the continuous locus of all such particles of the medium which are vibrating in the same phase at any instant.

Thus a wavefront is a surface of constant phase. The speed with which the wavefront moves outwards from the source is called the ***phase speed***.

Different types of wavefronts. The geometrical shape of a wavefront depends on the source of disturbance. Some of the common shapes are :

1. Spherical wavefront. In the case of waves travelling in all directions from a point source, the wavefronts are spherical in shape. This is because all such points which are equidistant from the point source will lie on a sphere [Fig. 10.1(*a*)] and the disturbance starting from the source *S* will reach all these points simultaneously.

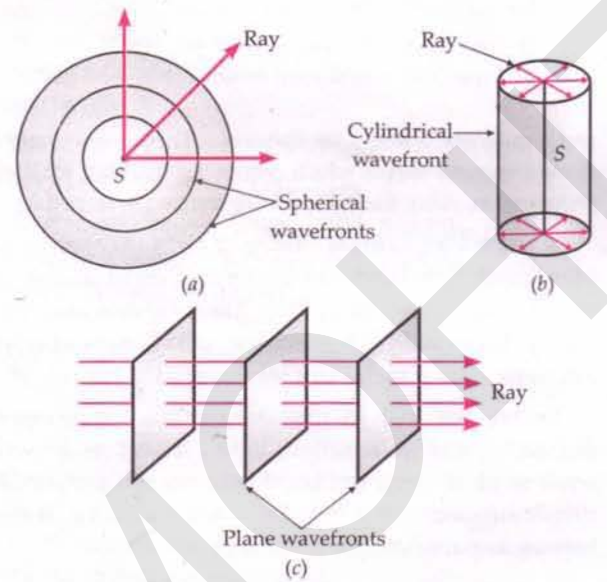


Fig. 10.1 Different types of wavefronts.

2. Cylindrical wavefront. When the source of light is linear in shape, such as a fine rectangular slit, the wavefront is cylindrical in shape. This is because the locus of all such points which are equidistant from the linear source will be a cylinder [Fig. 10.1(*b*)].

3. Plane wavefront. As a spherical or cylindrical wavefront advances, its curvature decreases progressively. So a small portion of such a wavefront at a large distance from the source will be a plane wavefront [Fig. 10.1(*c*)].

Ray of light. It is seen that whatever is the shape of a wavefront, the disturbance travels outwards along straight lines emerging from the source, *i.e.*, the energy of a wave travels in a direction perpendicular to the wavefront.

An arrow drawn perpendicular to a wavefront in the direction of propagation of a wave is called a ***ray***.

A ray of light represents the path along which light travels.

If we measure the separation between a pair of wavefronts along any ray, it is found to be a constant.

This illustrates two general principles :

1. *Rays are perpendicular to wavefronts.*
2. *The time taken for light to travel from one wavefront to another is the same along any ray.*

In case of a plane wavefront, the rays are parallel [Fig. 10.2(*a*)]. A group of parallel rays is called a ***beam of light***. In case of a spherical wavefront, the rays either converge to a point [Fig. 10.2(*b*)] or diverge from a point [Fig. 10.2(*c*)].

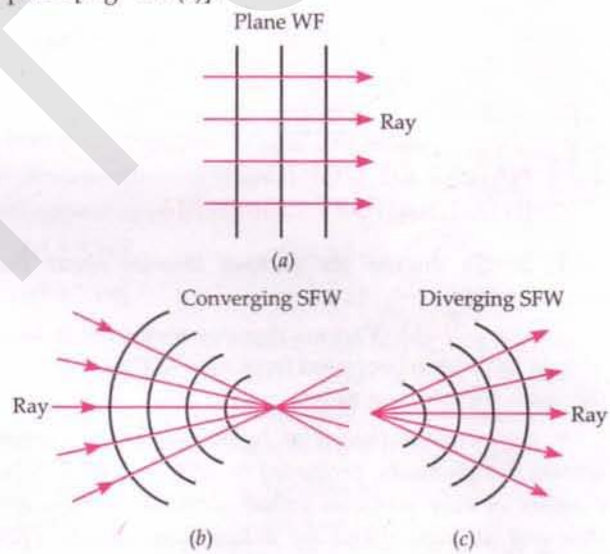


Fig. 10.2 Wavefronts and corresponding rays in three cases (*a*) plane, (*b*) converging spherical and (*c*) diverging spherical.

10.3 HUYGENS' PRINCIPLE OF SECONDARY WAVELETS

3. *State the assumptions on which Huygens' principle of secondary wavelets is based. Describe Huygens' construction for the propagation of wavefronts in a medium. Is a backward wavefront also possible ?*

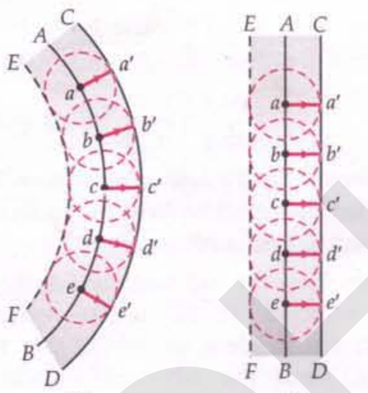
Huygens' principle. Huygens' principle is the basis of wave theory of light. It tells how a wavefront propagates through a medium. According to ***Huygens' principle****, each point on a wavefront is a source of secondary waves, which add up to give a wavefront at any later time.*

This principle is based on the following assumptions :

1. *Each point on a wavefront acts as a fresh source of new disturbance, called secondary waves or wavelets.*
2. *The secondary wavelets spread out in all directions with the speed of light in the given medium.*
3. *The new wavefront at any later time is given by the forward envelope (tangential surface in the forward direction) of the secondary wavelets at that time.*

Huygens' construction. It is a geometrical method of locating the new position and shape of a wavefront at any instant from its known position and shape at any other instant. The various steps involved are as follows :

1. Consider a spherical [Fig. 10.3(*a*)] or plane [Fig. 10.3(*b*)] wavefront moving towards right. Let *AB* be its position at any instant of time. The region on its left has received the wave while region on the right is undisturbed.



(*a*) (*b*)

Fig. 10.3 Huygens' geometrical construction for the propagation of (*a*) spherical, (*b*) plane wavefront.

2. According to Huygens' principle, each point on *AB* becomes a source of secondary disturbance, which travels with the same speed *c*. To find the new wavefront after time *t*, we draw spheres of radii *ct*, from each point on *AB*.

3. The forward envelope or the tangential surface *CD* of the secondary wavelets gives the new wavefront after time *t*.

4. The lines *aa'*, *bb'*, *cc'*, etc., are perpendicular to both *AB* and *CD*. Along these lines, the energy flows from *AB* to *CD*. So these lines represent the rays. Rays are always normal to wavefronts.

No backward wavefront is possible. There cannot be backward flow of energy during the propagation of a wave. It can be shown mathematically that the amplitude of secondary wavelets is proportional to $(1 + \cos\theta)$, where θ is the angle between the ray at the point of consideration and the direction of secondary wavelets. For a backward wavefront $\theta = \pi$, so that $1 + \cos\theta = 0$. Thus the resultant amplitude of all the secondary wavelets at any point on the backward wavefront is zero. In fact, the effects of secondary wavelets cancel out at all points except those lying on the forward envelope. So a backward wavefront cannot exist.

Huygens' principle of secondary wavelets can be used to prove the laws of reflection and refraction.

For Your Knowledge

- A wavefront is a surface of constant phase.
- A ray of light is the path along which light travels. It is always normal to the wavefront.
- In a homogeneous and isotropic medium (*i.e.*, a medium having uniform composition and the same properties in all directions), the speed of light is same in all directions and the secondary wavelets are spherical. The rays are then perpendicular to both the wavefronts and the time of travel measured along any ray from one wavefront to the next is always same.
- Only the front portions of the secondary wavelets add up to give rise to a wavefront in the forward direction. The back portions of the secondary wavelets add up to zero, so no backward wavefront is possible.

10.4 REFLECTION ON THE BASIS OF WAVE THEORY

4. *Derive the laws of reflection of light on the basis of Huygens' wave theory of light.*

Laws of reflection on the basis of Huygens' wave theory. As shown in Fig. 10.4, consider a plane wavefront *AB* incident on the plane reflecting surface *XY*, both the wavefront and the reflecting surface being perpendicular to the plane of paper.

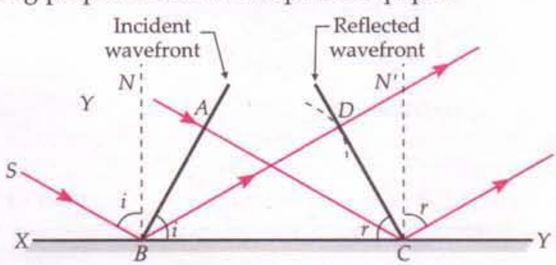


Fig. 10.4 Wavefronts and corresponding rays for reflection from a plane surface.

First the wavefront touches the reflecting surface at *B* and then at the successive points towards *C*. In accordance with Huygens' principle, from each point on *BC*, secondary wavelets start growing with the speed c. During the time the disturbance from A reaches the point C, the secondary wavelets from B must have spread over a hemisphere of radius $BD = AC = ct$, where t is the time taken by the disturbance to travel from A to C. The tangent plane CD drawn from the point C over this hemisphere of radius ct will be the *new reflected wavefront*.

Let angles of incidence and reflection be i and r respectively. In ΔABC and ΔDCB, we have

$$\angle BAC = \angle CDB \qquad \text{[Each is } 90^\circ]$$

$$BC = BC \qquad \text{[Common]}$$

$$AC = BD \qquad \text{[Each is equal to } ct]$$

$$\therefore \quad \Delta ABC \cong \Delta DCB$$

$$\text{Hence} \quad \angle ABC = \angle DCB$$

$$\text{or} \quad \angle i = \angle r$$

i.e., the angle of incidence is equal to the angle of reflection. This proves the first law of reflection.

Further, since the incident ray SB, the normal BN and the reflected ray BD are respectively perpendicular to the incident wavefront AB, the reflecting surface XY and the reflected wavefront CD (all of which are perpendicular to the plane of the paper), therefore, they all lie in the plane of the paper, *i.e.*, in the same plane. This proves the second law of reflection.

10.5 REFRACTION ON THE BASIS OF WAVE THEORY

5. *Derive the laws of refraction of light on the basis of Huygens' wave theory of light.*

Laws of refraction on the basis of Huygens' wave theory. Consider a plane wavefront AB incident on a plane surface XY, separating two media 1 and 2, as shown in Fig. 10.5. Let v_1 and v_2 be the velocities of light in the two media, with $v_2 < v_1$.

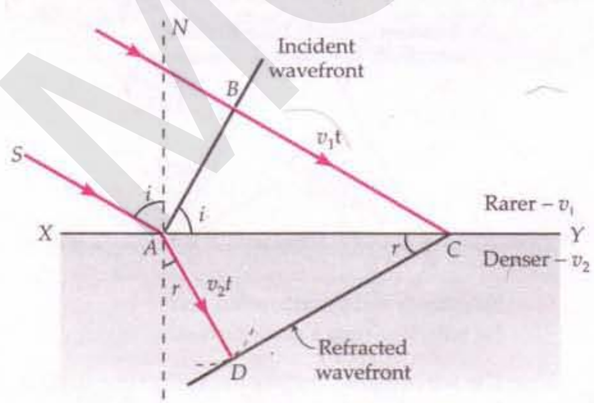


Fig. 10.5 Wavefronts and corresponding rays for refraction by a plane surface separating two media.

The wavefront first strikes at point A and then at the successive points towards C. According to Huygens' principle, from each point on AC, the secondary wavelets start growing in the second medium with speed v_2. Let the disturbance take time t to travel from B to C, then $BC = v_1 t$. During the time the disturbance from B reaches the point C, the secondary wavelets from point A must have spread over a hemisphere of radius $AD = v_2 t$ in the second medium. The tangent plane CD drawn from point C over this hemisphere of radius $v_2 t$ will be the new refracted wavefront.

Let the angles of incidence and refraction be i and r respectively.

From right ΔABC, we have

$$\sin \angle BAC = \sin i = \frac{BC}{AC}$$

From right ΔADC, we have

$$\sin \angle DCA = \sin r = \frac{AD}{AC}$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1 t}{v_2 t}$$

$$\text{or} \quad \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = {}^1\mu_2 \quad \text{(a constant)}$$

This proves ***Snell's law of refraction.*** The constant ${}^1\mu_2$ is called the *refractive index* of the second medium with respect to first medium.

Further, since the incident ray SA, the normal AN and the refracted ray AD are respectively perpendicular to the incident wavefront AB, the dividing surface XY and the refracted wavefront CD (all perpendicular to the plane of the paper), therefore, they all lie in the plane of the paper, *i.e.*, in the same plane. This proves another law of refraction.

Refraction at a rarer medium. Fig. 10.6 shows the refraction of a plane wavefront at a rarer medium *i.e.*,

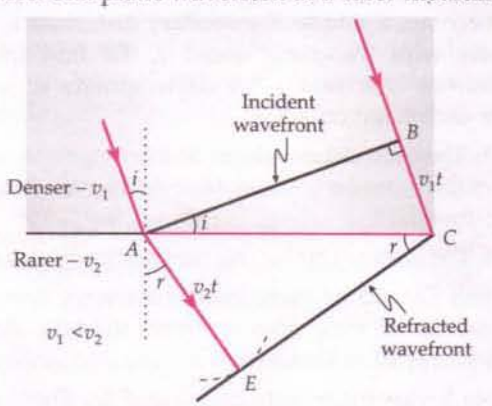


Fig. 10.6 Refraction of a plane wave incident on a rarer medium for which $v_2 > v_1$. The plane wave bends away from the refracting surface.

$v_2 > v_1$. The incident and refracted wavefronts are shown in Fig. 10.6. In this case, the angle of refraction is greater than the angle of incidence. Here also the Snell's law of refraction is valid. That is

$$\frac{\sin i}{\sin r} = {}^1\mu_2 \text{ (a constant)}$$

10.6 EFFECT ON WAVELENGTH, FREQUENCY AND SPEED DURING REFRACTION

6. *Discuss the effect on wavelength, frequency and speed of light as it passes from one medium to another.*

Effect on wavelength, frequency and speed during refraction. Consider a source of light at rest in one medium and the observer at rest in another medium. Let there be no relative motion between the two media so that the geometry of the source, medium and observer does not change with time. Then the light will take a definite time to travel from the source to the observer.

Suppose the source emits a wavefront after every time interval T and also each wavefront takes time T to travel from the source to the observer. Then the observer will receive $\nu = 1/T$ wavefronts per second. Thus *the frequency* ν *remains the same as light travels from one medium to another.* In fact, frequency ν is the characteristic of the source.

As the speeds of light v_1 and v_2 are different in the two media, the wavelengths λ_1 and λ_2 will also be different. Using the relation $v = \nu\lambda$, we get

$$\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\nu\lambda_1}{\nu\lambda_2} = \frac{\lambda_1}{\lambda_2}$$

Hence *the wavelength in a medium is directly proportional to the phase speed (or wave speed) and inversely proportional to its refractive index.*

NOTE The refractive index of a medium with respect to vacuum is

$$\mu = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} = \frac{c}{v}$$

Since any medium is optically denser than vacuum, so

$$\mu > 1$$

Consequently, $c > v$

Thus the speed of light in an optically rarer medium is greater than that in an optically denser medium.

10.7 BEHAVIOUR OF A PRISM, LENS AND MIRROR

7. *Discuss the action of a prism, a convex lens and a concave mirror, when a plane wavefront is incident on each of them.*

Behaviour of a prism. Fig. 10.7 shows the refraction of a plane wavefront through a thin prism. Since the speed of light in glass is smaller than that in air, therefore, the lower part C of the plane wavefront which travels through the greatest thickness of the glass prism is slowed down the most and the upper part A, which travels through the minimum thickness of the glass prism, is slowed down the least. This explains the tilting of a plane wavefront after refraction through a glass prism.

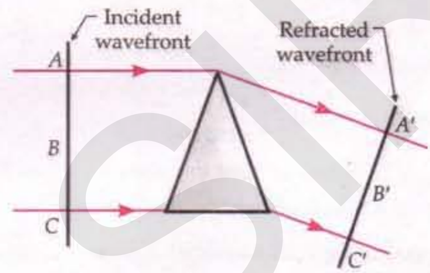


Fig. 10.7 Refraction of a plane wavefront through a prism.

Behaviour of a convex lens. Fig. 10.8 shows the refraction of a plane wavefront through a convex lens. The central part B of the plane wavefront travels through the greatest thickness of the lens and is, therefore, slowed down the most. The marginal parts A and C of the wavefront travel through a minimum thickness of the lens and are, therefore, slowed down the least. So the emerging wavefront is spherical and converges to a focus F.

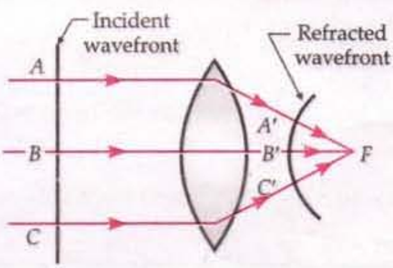


Fig. 10.8 Refraction of a plane wavefront through a convex lens.

Behaviour of a concave mirror. Fig. 10.9 shows the reflection of a plane wavefront from a concave mirror.

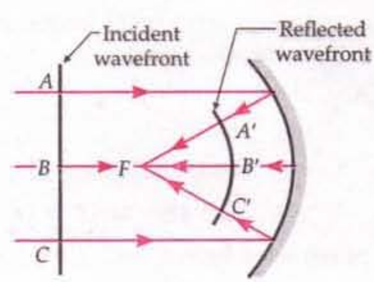


Fig. 10.9 Reflection of a plane wavefront from a concave mirror.

The central part B of the incident wavefront has to travel the greatest distance before getting reflected, compared to the marginal parts A and C. Therefore, the central portion B' of the reflected wavefront is closer to the mirror than the marginal portions A' and C'. Hence the reflected wavefront is spherical and converges to a focus.

Examples based on Reflection and Refraction of Light Waves

Formulae Used

1. Snell's law, $^1\mu_2 = \frac{\sin i}{\sin r}$
2. $\mu = \frac{c}{v} = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$
3. Speed of light in vacuum, $c = \nu\lambda$
4. $\mu = \frac{\lambda}{\lambda'} = \frac{\text{Wavelength in vacuum}}{\text{Wavelength in medium}}$
5. Wavelength in medium, $\lambda' = \frac{\lambda}{\mu}$
6. Optical path (in vacuum) $= \mu \times$ Path in medium
7. Frequency of light remains unchanged during its reflection or refraction.

Units Used

Speeds of light c and v are in ms^{-1}, wavelengths λ and λ' in metre, frequency ν in Hz and refractive index μ has no units.

Constant Used

Speed of light in vacuum, $c = 3 \times 10^8 ms^{-1}$

Conversions Used

$1 Å = 10^{-8}$ cm $= 10^{-10}$ m, 1 nm $= 10^{-9}$ m,

$1 \mu m = 10^{-6}$ m

Example 1. *Monochromatic light of wavelength 600 nm is incident from air on a glass surface. What are the wavelength, frequency and speed of refracted light ? Refractive index of glass 1.5.* [NCERT]

Solution. During refraction, frequency remains unchanged. Both wavelength and speed get changed.

Frequency,

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8 ms^{-1}}{600 \times 10^{-9} m}$$

$$= \mathbf{5 \times 10^{14}\ Hz}$$

Speed of refracted light,

$$v_{glass} = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = \mathbf{2 \times 10^8\ ms^{-1}}$$

Wavelength of refracted light,

$$\lambda_{glass} = \frac{v_{glass}}{\nu}$$

$$= \frac{2 \times 10^8}{5 \times 10^{14}} = 4 \times 10^{-7} m = \mathbf{400\ nm}.$$

Example 2. *The refractive index of diamond is 2.47 and that of window glass is 1.51. How much faster does light travel in window glass than in diamond ?*

Solution. Refractive index of diamond,

$$\mu_d = \frac{c}{v_d}$$

$$\therefore \quad v_d = \frac{c}{\mu_d} = \frac{3 \times 10^8}{2.47} = 1.215 \times 10^8 ms^{-1}$$

Refractive index of glass, $\mu_g = \frac{c}{v_g}$

$$\therefore \quad v_g = \frac{c}{\mu_g} = \frac{3 \times 10^8}{1.51}$$

$$= 1.987 \times 10^8 ms^{-1}$$

$$v_g - v_d = (1.987 - 1.215) \times 10^8$$

$$= \mathbf{7.72 \times 10^7\ ms^{-1}}$$

Thus light travels $7.72 \times 10^7 ms^{-1}$ faster in window glass than in diamond.

Example 3. *Calculate the time which light will take to travel normally through a glass plate of thickness 1 mm. Refractive index of glass is 1.5 and velocity of light is $3 \times 10^8 ms^{-1}$.*

Solution. Velocity of light in glass

$$= \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 ms^{-1}$$

Thickness of glass plate = 1 mm $= 10^{-3}$ m

$\therefore$ Time taken by light to pass normally through glass plate,

$$t = \frac{10^{-3}}{2 \times 10^8} = \mathbf{5 \times 10^{-12}\ s.}$$

Example 4. *White light consists of waves of wavelengths between 400 nm to 700 nm. What will be the wavelength range if this light goes through water ($\mu = 1.33$) ?*

Solution. When $\lambda = 400$ nm,

$$\lambda_w = \frac{\lambda}{\mu} = \frac{400}{1.33} = 300 \text{ nm.}$$

When $\lambda = 700$ nm,

$$\lambda_w = \frac{\lambda}{\mu} = \frac{700}{1.33} = 525 \text{ nm}$$

Thus the wavelength of white light in water varies from 300 nm to 525 nm.

Example 5. *The optical path of monochromatic light is the same if it travels 2.0 cm thickness of glass or 2.25 cm thickness of water. If the refractive index of water is 1.33, what is the refractive index of glass ?*

Solution. Optical path $= \mu \times$ Path in medium

$\because$ Optical path for glass = Optical path for water

$$\therefore \quad \mu_g \times 2.0 = 1.33 \times 2.25$$

or
$$\mu_g = \frac{1.33 \times 2.25}{2.0} = \mathbf{1.50.}$$

Example 6. *The number of waves in a 4 cm thick strip of glass is the same as in 5 cm thick water layer, when the same monochromatic light travels in them. If the refractive index of water is 4/3, what will be that of glass ?*

Solution. Number of waves in glass strip

$$= \frac{\text{Thickness of glass strip}}{\text{Wavelength in glass}} = \frac{4 \text{ cm}}{\lambda_g}$$

Number of waves in water layer

$$= \frac{\text{Thickness of water layer}}{\text{Wavelength in water}} = \frac{5 \text{ cm}}{\lambda_w}$$

As given,

$$\frac{4 \text{ cm}}{\lambda_g} = \frac{5 \text{ cm}}{\lambda_w} \quad \therefore \quad \frac{\lambda_g}{\lambda_w} = \frac{4}{5}$$

But wavelength of light in a medium is inversely related to its refractive index, therefore

$$\frac{\lambda_g}{\lambda_w} = \frac{\mu_w}{\mu_g} \quad \text{or} \quad \frac{4}{5} = \frac{\mu_w}{\mu_g}$$

$$\therefore \quad \mu_g = \frac{5}{4} \times \mu_w = \frac{5}{4} \times \frac{4}{3} = \mathbf{\frac{5}{3}}.$$

Example 7. *The absolute refractive index of air is 1.0003 and wavelength of yellow light in vacuum is 6000 Å. Find the thickness of air column which will contain one more wavelength of yellow light than in the same thickness of vacuum.*

Solution. Wavelength of yellow light in vacuum,

$$\lambda = 6000 \text{ Å}$$

Wavelength of yellow light in air,

$$\lambda' = \frac{\lambda}{\mu} = \frac{6000}{1.0003} \text{ Å}$$

Let a thickness t of vacuum contain n waves and the same thickness t of air contain $n+1$ waves.

Then
$$n = \frac{t}{\lambda} = \frac{t}{6000 \text{ Å}}$$

and
$$n + 1 = \frac{t}{\lambda'} = \frac{1.0003\, t}{6000 \text{ Å}}$$

From the above two equations, we get

$$\frac{t}{6000 \text{ Å}} + 1 = \frac{1.003\, t}{0.0003} \quad \text{or} \quad t + 6000 \text{ Å} = 1.0003\, t$$

or
$$t = \frac{6000}{0.0003} = 2 \times 10^7 \text{ Å} = \mathbf{2 \text{ mm}}.$$

Problems For Practice

1. The speed of the yellow light in a certain liquid is $2.4 \times 10^8 \text{ ms}^{-1}$. Find the refractive index of the liquid. (Ans. 1.25)
2. The wavelength range of the light that is visible to an average human being is 400 nm to 700 nm. What is the frequency range of this visible light ? (Ans. 4.3×10^{14} Hz to 7.5×10^{14} Hz)
3. The wavelength of light coming from a sodium source is 589 nm. What will be its wavelength in water ? Refractive index of water is 1.33. (Ans. 443 nm)
4. Light travels a certain distance in water in 3 μs. How much time it would take for light to travel the same distance in air ? Refractive index of water $= 4/3$. (Ans. 2.25 μs)
5. Red light of wavelength 750 nm enters a glass plate of refractive index 1.5. If the velocity of light in vacuum is $3 \times 10^8 \text{ ms}^{-1}$, calculate in the glass (*i*) frequency (*ii*) velocity and (*iii*) wavelength of light. **[Punjab 94C]**
   [Ans. (*i*) 4×10^{14} Hz (*ii*) $2 \times 10^8 \text{ ms}^{-1}$ (*iii*) 500 nm]
6. The absolute refractive indices of glass and water are 3/2 and 4/3. Determine the ratio of the speeds of light in glass and water. (Ans. 8 : 9)
7. The refractive index of glass with respect to water is 1.125. If the speed of light in water is $2.25 \times 10^8 \text{ ms}^{-1}$ then calculate the speed of light in glass. (Ans. $2.0 \times 10^8 \text{ ms}^{-1}$)
8. The refractive index of glass is 1.5 and that of water is 1.3, the speed of light in water is $2.25 \times 10^8 \text{ ms}^{-1}$. What is the speed of light in glass ? **[ISCE 96]** (Ans. $1.95 \times 10^8 \text{ ms}^{-1}$)
9. The ratio of the thickness of the strips of two transparent media *A* and *B* is 3 : 2. If light takes the same time in passing through both of them, then what is the refractive index of *B* with respect to *A* ? (Ans. 1.5)
10. The speed of light in air is $3 \times 10^8 \text{ ms}^{-1}$. If refractive index of glass is 1.5, find the time taken by light to travel a distance of 10 cm in glass. (Ans. 5×10^{-10} s)
11. A light wave has a frequency of 5×10^{14} Hz. Find the difference in its wavelengths in alcohol of refractive index 1.35 and glass of refractive index 1.5. (Ans. 445 Å)

HINTS

1. $\mu_l = \frac{c}{v_l} = \frac{3 \times 10^8}{2.4 \times 10^8} = \mathbf{1.25.}$

2. $\nu_1 = \frac{c}{\lambda_1} = \frac{3 \times 10^8}{400 \times 10^{-9}} = \mathbf{7.5 \times 10^{14}\ Hz}$;

   $\nu_2 = \frac{c}{\lambda_2} = \frac{3 \times 10^8}{700 \times 10^{-9}} = \mathbf{4.3 \times 10^{14}\ Hz.}$

3. Here $\lambda = 589$ nm, $\mu = 1.33$

   Wavelength in water, $\lambda_w = \frac{\lambda}{\mu} = \frac{589}{1.33} = \mathbf{443\ nm.}$

4. $\mu = \frac{v_a}{v_l} = \frac{d/t_a}{d/t_l} = \frac{t_l}{t_a} \quad \therefore \quad \frac{4}{3} = \frac{3\,\mu s}{t_a}$

   or $t_a = \frac{3}{4/3} = \mathbf{2.25\ \mu s.}$

5. Proceed as in Example 1 on page 10.6.

6. $v \propto \frac{1}{\mu} \quad \therefore \quad \frac{v_g}{v_w} = \frac{\mu_w}{\mu_g} = \frac{4/3}{3/2} = \mathbf{8 : 9}$

7. ${}^w\mu_g = \frac{\text{Speed of light in water}}{\text{Speed of light in glass}}$

   $\therefore \quad 1.125 = \frac{2.25 \times 10^8\ \text{ms}^{-1}}{v_g}$

   or $\quad v_g = \frac{2.25 \times 10^8}{1.125} = \mathbf{2.0 \times 10^8\ ms^{-1}.}$

8. $\frac{v_g}{v_w} = \frac{\mu_w}{\mu_g}$ or $v_g = \frac{\mu_w}{\mu_g} \times v_w = \frac{1.3}{1.5} \times 2.25 \times 10^8$

   $= \mathbf{1.95 \times 10^8\ ms^{-1}.}$

9. Let the thickness of the two strips be $3x$ and $2x$ and velocities of light through them be v_A and v_B. As the light takes same time in passing through them, therefore

   $$\frac{3x}{v_A} = \frac{2x}{v_B} \quad \text{or} \quad \frac{v_A}{v_B} = \frac{3}{2}$$

   Hence ${}^A\mu_B = \frac{v_A}{v_B} = \frac{3}{2} = \mathbf{1.5.}$

10. $v_g = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8\ \text{ms}^{-1}$

    Time, $t = \frac{\text{Displacement}}{v_g} = \frac{10 \times 10^{-2}}{2 \times 10^8} = \mathbf{5 \times 10^{-10}\ s.}$

11. Wavelength in vacuum,

    $$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{5 \times 10^{14}} = 6 \times 10^{-7}\ \text{m}$$

    $$\lambda_{alc} = \frac{\lambda}{\mu_{alc}} = \frac{6 \times 10^{-7}}{1.35} = 4445\ \text{Å},$$

    $$\lambda_g = \frac{\lambda}{\mu_g} = \frac{6 \times 10^{-7}}{1.5} = 4000\ \text{Å}$$

    $$\lambda_{alc} - \lambda_g = 4445 - 4000 = \mathbf{445\ Å.}$$

10.8 PRINCIPLE OF SUPERPOSITION OF WAVES

8. *State and explain the principle of superposition of waves.*

Principle of superposition of waves. When a number of waves travel through a medium simultaneously, each wave travels independently of the others *i.e.*, as if all other waves were absent. An important consequence of this independent behaviour of the waves is that the effects of all these waves get added together. The resultant wave is obtained by the ***principle of superposition of waves*** which can be stated as follows :

> *When a number of waves travelling through a medium superpose on each other, the resultant displacement at any point at a given instant is equal to the vector sum of the displacements due to the individual waves at that point.*

If $\vec{y}_1, \vec{y}_2, \vec{y}_3, \ldots, \vec{y}_n$ are the displacements due to the different waves acting separately, then according to the principle of superposition, the resultant displacement when all the waves act together is given by the vector sum :

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \ldots + \vec{y}_n$$

When the two superposing waves are in the same phase *i.e.*, the crest of one falls over the crest of another [Fig. 10.10(*a*)] or the trough of one falls over the trough of another [Fig. 10.10(*b*)], their displacements get added. When the two waves meet in opposite phases *i.e.*, the crest of one falls over the trough of another [Fig. 10.10(*c*)], their displacements get subtracted.

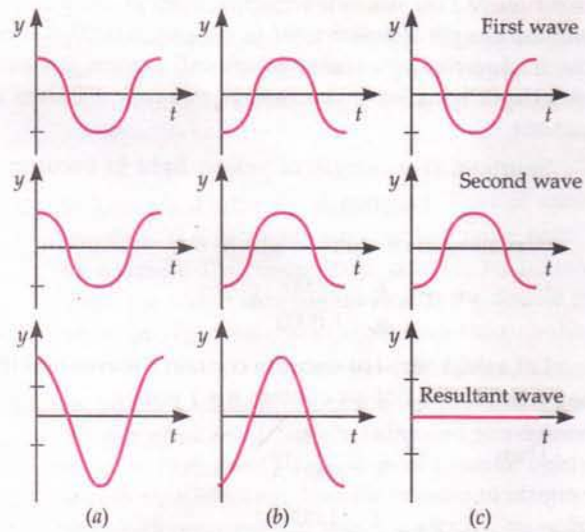


Fig. 10.10 Illustration of principle of superposition of waves.

10.9 INTERFERENCE OF LIGHT : YOUNG'S DOUBLE SLIT EXPERIMENT

9. *What is interference of light ? Describe Young's double slit experiment for observing the interference of light.*

Interference of light. *When two light waves of the same frequency and having zero or constant phase difference travelling in the same direction superpose each other, the intensity in the region of superposition gets redistributed, becoming maximum at some points and minimum at others. This phenomenon is called* ***interference of light****.*

Young's double slit experiment. In 1801, *Thomas Young* was the first person to demonstrate experimentally the interference of light.

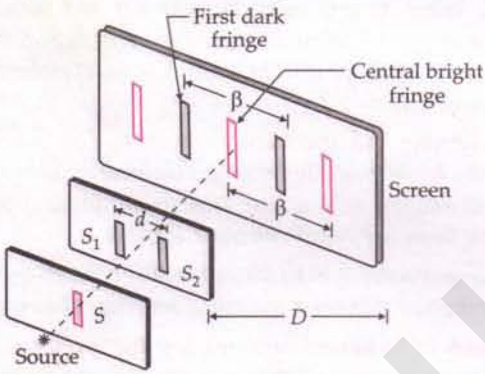


Fig. 10.11 Young's double slit experiment.

In this experiment, a source of monochromatic light (*e.g.*, a sodium vapour lamp) illuminates a rectangular narrow slit *S*, about 1 mm wide, as shown in Fig. 10.11. S_1 and S_2 are two parallel narrow slits which are arranged symmetrically and parallel to the slit *S* at a distance of about 10 cm from it. The separation between S_1 and S_2 is ≈ 2 mm and width of each slit is ≈ 0.3 mm. An observation screen is placed at a distance of ≈ 2 m from the two slits. Alternate bright and dark bands appear on the observation screen. These are called ***interference fringes***. When one of the slits, S_1 or S_2 is closed, bright and dark fringes disappear and the intensity of light becomes uniform.

Explanation. Fig. 10.12 shows a section of Young's experiment in the plane of paper. According to Huygens' principle, cylindrical wavefronts emerge out from slit *S*, whose sections have been shown by circular arcs. The solid curves represent crests and the dotted curves represent troughs. As $SS_1 = SS_2$, these waves fall on the slits S_1 and S_2 simultaneously so that the waves spreading out from S_1 and S_2 are in the same phase. Thus S_1 and S_2 act as two *coherent sources* of monochromatic light. Interference takes place between the waves diverging from these sources.

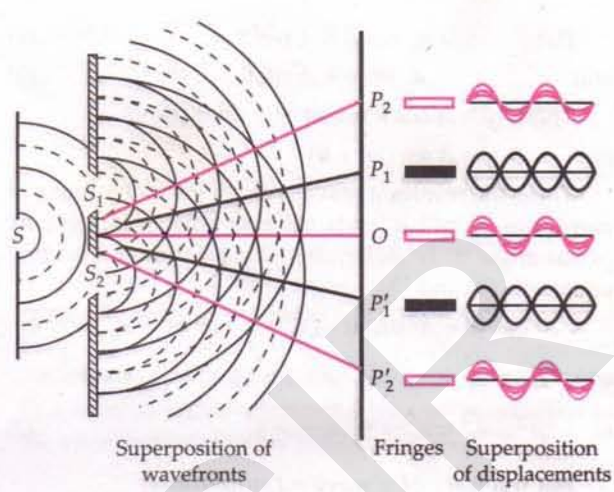


Fig. 10.12 Interference of two light beams.

At the lines leading to *O*, P_2 and P_2', the crest of one wave falls over the crest of other wave or the trough of one wave falls over the trough of other wave, the amplitudes of the two waves get added up and hence the intensity ($I \propto a^2$) becomes maximum. This is called ***constructive interference***. At the lines leading to P_1 and P_1', the crest of one wave falls over the trough of other or the trough of one wave falls over the crest of other wave, the amplitudes of the two waves get subtracted and hence the intensity becomes minimum. This is called ***destructive interference***. So on the observation screen, we obtain a number of alternate bright and dark fringes, parallel to the two slits.

10.10 CONDITIONS FOR CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE

10. *Derive an expression for the intensity at any point on the observation screen in Young's double slit experiment. Hence write the conditions for constructive and destructive interference.*

Expression for intensity at any point in interference pattern. Suppose the displacements of two light waves from two coherent sources S_1 and S_2 at point *P* on the observation screen at any time *t* are given by

$$y_1 = a_1 \sin \omega t$$

and $$y_2 = a_2 \sin (\omega t + \phi)$$

where a_1 and a_2 are the amplitudes of the two waves, ϕ is the constant phase difference between the two waves. By the superposition principle, the resultant displacement at point *P* is

$$y = y_1 + y_2 = a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$

or $$y = (a_1 + a_2 \cos \phi) \sin \omega t + a_2 \sin \phi \cos \omega t$$

Put $a_1 + a_2 \cos\phi = A\cos\theta$ ...(1)

and $a_2 \sin\phi = A\sin\theta$ ...(2)

Then $y = A\cos\theta \sin\omega t + A\sin\theta\cos\omega t$

or $y = A\sin(\omega t + \theta)$

Thus, the resultant wave is also a harmonic wave of amplitude A and it leads the first harmonic wave by phase angle θ. To determine A, squaring and adding equations (1) and (2), we get

$$A^2\cos^2\theta + A^2\sin^2\theta = (a_1 + a_2\cos\phi)^2 + a_2^2\sin^2\phi$$

or $A^2 = a_1^2 + a_2^2(\cos^2\phi + \sin^2\phi) + 2a_1a_2\cos\phi$

or $A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$ ...(3)

But intensity of a wave $\propto$ (amplitude)^2

We write $I = kA^2,\ I_1 = ka_1^2$ and $I_2 = ka_2^2$

where k is proportionality constant. The equation (3) can be written as

$$kA^2 = ka_1^2 + ka_2^2 + 2\sqrt{k}a_1\sqrt{k}a_2\cos\phi$$

or $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$ ...(4)

This equation gives the total intensity at a point where the phase difference is ϕ. Here I_1 and I_2 are the intensities which the two individual sources produce on their own. The total intensity also contains a third term $2\sqrt{I_1I_2}\cos\phi$. It is called *interference term*.

Constructive interference. The resultant intensity at the point P will be maximum when

$$\cos\phi = 1 \quad \text{or} \quad \phi = 0, 2\pi, 4\pi, ...$$

Since a phase difference of 2π corresponds to a path difference of λ, therefore, if p is the path difference between the two superposing waves, then

$$\frac{2\pi p}{\lambda} = 0,\ 2\pi,\ 4\pi, ...$$

or $p = 0,\ \lambda,\ 2\lambda,\ 3\lambda, ... = n\lambda$

Hence the resultant intensity at a point is maximum when the phase difference between the two superposing waves is an even multiple of π or path difference is an integral multiple of wavelength λ. This is the condition of constructive interference.

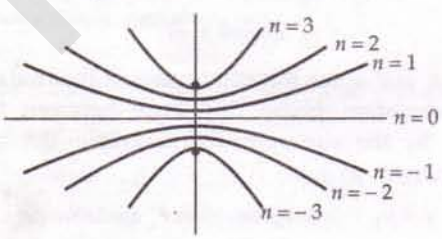


Fig. 10.13 Locus of points for which $S_1P - S_2P$ is equal to $0, \pm\lambda, \pm 2\lambda, \pm 3\lambda$.

Destructive interference. The resultant intensity at the point P will be minimum when

$$\cos\phi = -1 \quad \text{or} \quad \phi = \pi,\ 3\pi,\ 5\pi, ...$$

or $\dfrac{2\pi p}{\lambda} = \pi,\ 3\pi,\ 5\pi, ...$

or $p = \dfrac{\lambda}{2}, \dfrac{3\lambda}{2}, \dfrac{5\lambda}{2}, ... = (2n-1)\dfrac{\lambda}{2}$

Hence the resultant intensity at a point is minimum when the phase difference between the two superposing waves is an odd multiple of π or the path difference is an odd multiple of $\lambda/2$. This is the condition of destructive interference.

10.11 COHERENT & INCOHERENT SOURCES

11. *What do you mean by coherent and incoherent sources of light ? Why are coherent sources required to produce interference of light ? Can two independent light sources be coherent ?*

Coherent and incoherent sources. *Two sources of light which continuously emit light waves of same frequency (or wavelength) with a zero or constant phase difference between them, are called **coherent sources**.*

*Two sources of light which do not emit light waves with a constant phase difference are called **incoherent sources**.*

Need of coherent sources for the production of interference pattern. When two monochromatic waves of intensity I_1, I_2 and phase difference ϕ meet at a point, the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

The last term $2\sqrt{I_1I_2}\cos\phi$ is called ***interference term***. There are *two* possibilities :

1. If $\cos\phi$ remains constant with time, the total intensity at any point will be constant. The intensity will be maximum $(\sqrt{I_1} + \sqrt{I_2})^2$ at points where $\cos\phi$ is $+1$ and minimum $(\sqrt{I_1} - \sqrt{I_2})^2$ at points where $\cos\phi = -1$. The sources in this case are coherent.

2. If $\cos\phi$ varies continuously with time assuming both positive and negative values, then the average value of $\cos\phi$ will be zero over time interval of measurement. Then interference term averages to zero. There will be same intensity, $I = I_1 + I_2$ at every point *i.e.*, there will be general illumination on the observation screen. The two sources in the case are incoherent.

Hence to observe interference, we need to have two sources with the same frequency and with a stable phase difference. Such a pair of sources are called ***coherent sources***.

Two independent sources cannot be coherent. This is because of the following reasons :

1. Light is emitted by individual atoms and not by the bulk of matter acting as a whole.
2. Even a tiniest source consists of millions of atoms, and emission of light by them takes place independently.
3. Even an atom emits an unbroken wave of about 10^{-8} second due to its transition from a higher energy state to a lower energy state.

The millions of atoms of a source cannot emit waves in the same phase. The light emitted by the commonly used monochromatic source (a sodium lamp) remains coherent for about 10^{-8} s. After this time, the atoms responsible for emission of light get changed. The phase difference and hence the interference pattern changes 10^8 times in one second. Our eyes cannot see such rapid changes and a uniform illumination is seen on the screen. So two independent light sources cannot produce a sustained interference.

Two coherent sources can be obtained from a single parent source. Some of the methods of producing coherent sources are as follows :

1. In *Young's double slit experiment*, the two sources S_1 and S_2 get light from the same source S. Whatever phase changes occur in S_1, the same phase changes occur in S_2. The relative phase difference between S_1 and S_2 remains constant with time. So they act as coherent sources.
2. In *Fresnel's biprism method*, two coherent sources are obtained from the same parent source, by refraction.
3. In *Lloyd's mirror method*, a source and its reflected image act as two coherent sources.

12. *State the conditions, which must be satisfied for two light sources to be coherent.*

Conditions for obtaining two coherent sources of light :

1. *The two sources of light must be obtained from a single source by some method.* Then the relative phase difference between the two light waves from the sources will remain constant with time.

2. *The two sources must give monochromatic light.* Otherwise, different colours will produce different interference patterns and fringes of different colours will overlap.

3. *The path difference between the waves arriving on the screen from the two sources must not be large.* It should not exceed 30 cm. Then the phase difference produced due to path difference will not be constant. There will be general illumination on the screen.

For Your Knowledge

- To observe interference of light, the two sources of light must be coherent.
- In contrast to light from an ordinary source, the laser light is highly monochromatic and coherent. So *two independent laser sources can produce interference fringes* and the path difference may be several metres in this case.
- **Methods of producing coherent source.** There are two general methods of producing coherent sources :

  1. ***By division of wavefront.*** In this method, a wavefront is divided into two or more parts by use of slits, mirrors, lenses or prisms. *For example*, Young's double slit method, Frenel's biprism and Lloyd's mirror.

  2. ***By division of amplitude.*** Here the amplitude of the wave is divided into two or more parts by partial reflection or refraction. The divided parts travel along different paths and are made to superpose to produce interference. *For example*, the brilliant colours seen in thin films of transparent materials like soap film, oil film, etc.

Examples based on

Amplitude and Intensity at any point in an Interference Pattern

Formulae Used

1. Resultant amplitude, $a = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$
2. Resultant intensity, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$
3. When $I_1 = I_2 = I_0$, $I = 2I_0(1+\cos\phi) = 4I_0\cos^2\frac{\phi}{2}$.

Units Used

Amplitudes a, a_1 and a_2 are in metre and intensities I, I_1 and I_2 in watt / m^2.

Example 8. *Two plane monochromatic waves propagating in the same direction with amplitudes A and 2 A and differing in phase by π/3 rad superpose. Calculate the amplitude of the resultant wave.* [CBSE Sample Paper 03]

Solution. Here $A_1 = A$, $A_2 = 2A$, $\phi = \pi/3$

$$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$= \sqrt{A^2 + (2A)^2 + 2A \times 2A\cos\frac{\pi}{3}}$$

$$= \sqrt{5A^2 + 4A^2 \times \frac{1}{2}} = \sqrt{7A^2} = \sqrt{7}\,A$$

Example 9. *Two sources of intensity I and 4 I are used in an interference experiment. Find the intensity at points where the waves from two sources superimpose with a phase difference (i) zero (ii) π/2 and (iii) π.* [CBSE D 95C]

Solution. The resultant intensity at a point where phase difference is ϕ is

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

As $\quad I_1 = I$ and $I_2 = 4I$, therefore

$$I_R = I + 4I + 2\sqrt{I.4I}\cos\phi = 5I + 4I\cos\phi$$

(*i*) When $\phi = 0$, $I_R = 5I + 4I\cos 0 = \mathbf{9I}$.

(*ii*) When $\phi = \frac{\pi}{2}$, $I_R = 5I + 4I\cos\frac{\pi}{2} = \mathbf{5I}$.

(*iii*) When $\phi = \pi$,

$$I_R = 5I + 4I\cos\pi = 5I - 4I = \mathbf{I}.$$

Example 10. *In a Young's double slit experiment, the intensity of light at a point on the screen where the path difference is λ is k units. Find the intensity at a point where the path difference is (i) $\frac{\lambda}{4}$ (ii) $\frac{\lambda}{3}$ and (iii) $\frac{\lambda}{2}$.* [CBSE D 12, 14]

Solution. Intensity at any point on the screen,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Let I_0 be the intensity of either source. Then $I_1 = I_2 = I_0$, and

$$I = 2I_0(1 + \cos\phi) = 4I_0\cos^2\frac{\phi}{2}$$

When $p = \lambda$, $\phi = 2\pi$

$$\therefore \quad I = 4I_0\cos^2\frac{\phi}{2} = 4I_0\cos^2\pi = 4I_0 = k$$

(*i*) When $p = \frac{\lambda}{4}$, $\phi = \frac{\pi}{2}$

$$\therefore \quad I = 4I_0\cos^2\frac{\pi}{4} = 4I_0 \times \frac{1}{2} = 2I_0 = \mathbf{\frac{k}{2}}.$$

(*ii*) When $p = \frac{\lambda}{3}$, $\phi = \frac{2\pi}{3}$

$$\therefore \quad I = 4I_0\cos^2\frac{\pi}{3} = 4I_0 \times \frac{1}{4} = I_0 = \mathbf{\frac{k}{4}}.$$

(*iii*) When $p = \frac{\lambda}{2}$, $\phi = \pi$,

$$\therefore \quad I = 4I_0\cos^2\frac{\pi}{2} = \mathbf{0}.$$

Example 11. *Find the ratio of the intensity at the centre of a bright fringe to the intensity at a point one-quarter of the distance between two fringes from the centre.*

Solution. If I_0 is the intensity of either source, then intensity at a point is given by

$$I = 2I_0(1 + \cos\phi)$$

At the centre $\phi = 0$, then intensity will be

$$I_1 = 2I_0(1 + \cos 0) = 4I_0$$

The phase difference between two successive fringes is 2π. So the phase difference at a point distant one-quarter of the distance between two fringes from the centre will be $\pi/2$. Hence intensity at this point will be

$$I_2 = 2I_0\left(1 + \cos\frac{\pi}{2}\right) = 2I_0$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{4I_0}{2I_0} = \mathbf{2:1}$$

Example 12. *Find the ratio of intensities of two points P and Q on a screen in a Young's double slit experiment when waves from sources S_1 and S_2 have phase difference of (i) $\frac{\pi}{3}$ and (ii) $\frac{\pi}{2}$ respectively.* [CBSE OD 2000C]

Solution. As $I = 2I_0(1 + \cos\phi)$

$$\therefore \quad I \propto 1 + \cos\phi$$

Hence $\quad \frac{I_P}{I_Q} = \frac{1 + \cos\frac{\pi}{3}}{1 + \cos\frac{\pi}{2}} = \frac{1 + \frac{1}{2}}{1 + 0} = \frac{3}{2} = \mathbf{3:2}.$

Example 13. *Find the ratio of intensities at two points in a screen in Young's double slit experiment, when waves from the two slits have path difference of (i) 0 and (ii) $\lambda/4$.* [CBSE OD 03]

Solution. Intensity at any point of an interference pattern is given by

$$I = 2I_0(1 + \cos\phi)$$

where I_0 is the intensity of either wave.

Here $\phi_P = 0$,

$$\phi_Q = \frac{2\pi p}{\lambda} = \frac{2\pi}{\lambda}.\frac{\lambda}{4} = \frac{\pi}{2}$$

$$\therefore \quad \frac{I_P}{I_Q} = \frac{1 + \cos\phi_P}{1 + \cos\phi_Q} = \frac{1 + \cos 0}{1 + \cos\pi/2}$$

$$= \frac{1 + 1}{1 + 0} = \frac{2}{1} = \mathbf{2:1}.$$

Problems For Practice

1. The phase difference between two light waves reaching a point is $\pi/2$. What is the resultant amplitude if the individual amplitudes are 3 mm and 4 mm ? (Ans. 5 mm)
2. Two light waves superposing at the midpoint of the screen are coming from coherent sources of light of phase difference 3π radian. Their amplitudes are 1 cm each. What will be the resultant amplitude at the given point ? (Ans. zero)
3. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. What will be the maximum and minimum possible intensities ? [IIT] (Ans. $9I$, I)

4. In Young's double slit experiment, what is the intensity at a point on screen where the two waves arrive having a phase difference of (*i*) 60° (*ii*) 90° and (*iii*) 120°? Assume that intensity of each source is I_0. (Ans. $3I_0, 2I_0, I_0$)
5. Find the ratio of intensities of two points P and Q on a screen in Young's double slit experiment when waves from sources S_1 and S_2 have phase difference of (*i*) 0° and (*ii*) $\pi/2$ respectively. **[CBSE OD 2000C]** (Ans. 2 : 1)

HINTS

1. Here $a_1 = 3$ mm, $a_2 = 4$ mm, $\phi = \frac{\pi}{2}$

Resultant amplitude,

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$$
$$= \sqrt{3^2 + 4^2 + 2\times3\times4\cos\pi/2} = \mathbf{5\ mm.}$$

2. Here $a_1 = a_2 = 1$ cm, $\phi = 3\pi$

$$\therefore \quad a = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$$
$$= \sqrt{1^2 + 1^2 + 2\times1\times1\times\cos 3\pi}$$
$$= \sqrt{2 + 2\times(-1)} = \mathbf{0}.$$

3. Here $I_1 = I$ and $I_2 = 4I$.

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

When $\phi = 0$, intensity is maximum.

$$\therefore \quad I_{max} = I + 4I + 2\sqrt{I \cdot 4I}\cos 0 = 5I + 4I = \mathbf{9I}.$$

When $\phi = \pi$, intensity is minimum.

$$\therefore \quad I_{min} = I + 4I + 2\sqrt{I \cdot 4I}\cos\pi = 5I - 4I = \mathbf{I}.$$

4. The intensity at any point on the screen is

$$I = 4I_0\cos^2\frac{\phi}{2}$$

(*i*) When $\phi = 60°$,

$$I = 4I_0\cos^2 30° = 4I_0\left(\frac{\sqrt{3}}{2}\right)^2 = \mathbf{3I_0}$$

(*ii*) When $\phi = 90°$,

$$I = 4I_0\cos^2 45° = 4I_0\left(\frac{1}{\sqrt{2}}\right)^2 = \mathbf{2I_0}$$

(*iii*) When $\phi = 120°$,

$$I = 4I_0\cos^2 60° = 4I_0\left(\frac{1}{2}\right)^2 = \mathbf{I_0}.$$

5. As $I \propto 1 + \cos\phi$

$$\therefore \quad \frac{I_P}{I_Q} = \frac{1+\cos 0°}{1+\cos\pi/2} = \frac{1+1}{1+0} = \mathbf{2:1}.$$

10.12 THEORY OF INTERFERENCE FRINGES : FRINGE WIDTH

13. *Deduce an expression for fringe width in Young's double slit experiment. How can the wavelength of monochromatic light be found by this experiment ?*

Expression for fringe width in Young's double slit experiment. As shown in Fig. 10.14, suppose a narrow slit S is illuminated by monochromatic light of wavelength λ. S_1 and S_2 are two narrow slits at equal distance from S. Being derived from the same parent source S, the slits S_1 and S_2 act as two *coherent sources*, separated by a small distance d. Interference fringes are obtained on a screen placed at distance D from the sources S_1 and S_2.

Consider a point P on the screen at distance x from the centre O. The nature of the interference at the point P depends on path difference,

$$p = S_2P - S_1P$$

From right-angled ΔS_2BP and ΔS_1AP,

$$S_2P^2 - S_1P^2 = [S_2B^2 + PB^2] - [S_1A^2 + PA^2]$$
$$= \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right]$$

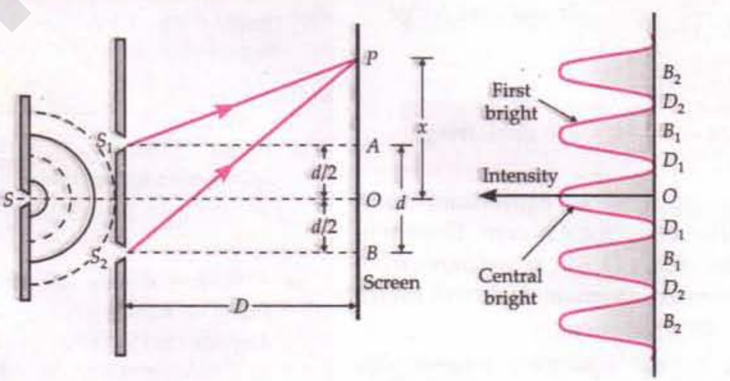


Fig. 10.14 Position of bright and dark fringes in Young's double slit experiment.

or $$(S_2P - S_1P)(S_2P + S_1P) = 2xd$$

or $$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

In practice, the point P lies very close to O, therefore $S_1P \simeq S_2P \simeq D$. Hence

$$p = S_2P - S_1P = \frac{2\,xd}{2\,D}$$

or $$p = \frac{xd}{D}$$

Positions of bright fringes. For constructive interference,

$$p = \frac{xd}{D} = n\lambda$$

or $$x = \frac{nD\lambda}{d} \quad \text{where } n = 0, 1, 2, 3, ...$$

Clearly, the positions of various bright fringes are as follows :

For $n = 0$, $x_0 = 0$ *Central bright fringe*

For $n = 1$, $x_1 = \frac{D\lambda}{d}$ *First bright fringe*

For $n = 2$, $x_2 = \frac{2D\lambda}{d}$ *Second bright fringe*

....

For $n = n$, $x_n = \frac{nD\lambda}{d}$ **nth bright fringe**

Positions of dark fringes. For destructive interference,

$$p = \frac{xd}{D} = (2n-1)\frac{\lambda}{2}$$

or $$x = (2n-1)\frac{D\lambda}{2d} \quad \text{where } n = 1, 2, 3, ...$$

Clearly, the positions of various dark fringes are as follows :

For $n = 1$, $x'_1 = \frac{1}{2}\frac{D\lambda}{d}$ *First dark fringe*

For $n = 2$, $x'_2 = \frac{3}{2}\frac{D\lambda}{d}$ *Second dark fringe*

....

For $n = n$, $x'_n = (2n-1)\frac{D\lambda}{2d}$ ***nth dark fringe***

Since the central point O is equidistant from S_1 and S_2, the path difference p for it is zero. There will be a bright fringe at the centre O. But as we move from O upwards or downwards, alternate dark and bright fringes are formed.

Fringe width. *It is the separation between two successive bright or dark fringes,*

Width of a dark fringe = Separation between two consecutive bright fringes

$$= x_n - x_{n-1} = \frac{nD\lambda}{d} - \frac{(n-1)\,D\lambda}{d} = \frac{D\lambda}{d}$$

Width of a bright fringe

= Separation between two consecutive dark fringes

$$= x'_n - x'_{n-1}$$

$$= (2n-1)\frac{D\lambda}{2d} - [2\,(n-1)-1]\frac{D\lambda}{2d} = \frac{D\lambda}{d}$$

Clearly, both the bright and dark fringes are of equal width.

Hence the expression for the fringe width in Young's double slit experiment can be written as

$$\beta = \frac{D\lambda}{d}$$

As β is independent of n (the order of fringe), therefore, all the fringes are of equal width. In the case of light, λ is extremely small, D should be much larger than d, so that the fringe width β may be appreciable and hence observable.

Measurement of wavelength. Young's double slit experiment can be used to determine the wavelength of a monochromatic light. The interference pattern is obtained in the focal plane of a micrometer eyepiece and with its help fringe width β is measured. By measuring the distance d between the two coherent sources and their distance D from the eyepiece, the value of wavelength λ can be calculated as

$$\lambda = \frac{\beta d}{D}$$

For Your Knowledge

- In Young's double slit experiment, the width of the central bright fringe is equal to the distance between the first dark fringes on the two sides of the central bright fringe. So the ***width of the central bright fringe*** is given by
$$\beta_0 = 2x'_1 = 2 \times \frac{D\lambda}{2d} = \frac{D\lambda}{d}$$
- As all the bright and dark fringes are of the same width, the ***angular width of a fringe*** is given by
$$\theta = \frac{\beta}{D} = \frac{D\lambda/d}{D} = \frac{\lambda}{d}$$
- If ***Young's double slit apparatus is immersed in a liquid*** of refractive index μ, the wavelength of light decreases to $\lambda'(= \lambda/\mu)$ and so the fringe width reduces to $$\beta' = \frac{D\lambda'}{d} = \frac{D\lambda}{\mu d} = \frac{\beta}{\mu}$$

Examples based on
Young's Double Slit Experiment

Formulae Used

1. For a bright fringe, path difference, $p = n\lambda$
2. For a dark fringe, $p = (2n-1)\dfrac{\lambda}{2}$, $n = 1, 2, 3, ...$
3. Distance of nth bright fringe from the centre of the screen,
$$x_n = n\frac{D\lambda}{d}, \quad n = 1, 2, 3, ...$$
4. Distance of nth dark fringe from the centre of the screen,
$$x'_n = (2n-1)\frac{D\lambda}{2d}$$
5. Fringe width, $\beta = \dfrac{D\lambda}{d}$
6. Wavelength of light used, $\lambda = \dfrac{\beta d}{D}$
7. Angular position of nth bright fringe,
$$\theta_n = \frac{x_n}{D} = \frac{n\lambda}{d}.$$
8. Angular position of nth dark fringe,
$$\theta'_n = \frac{x'_n}{D} = (2n-1)\frac{\lambda}{2d}.$$

Units Used

Path difference p, distances x_n, x'_n, d and D; wavelength λ and fringe width β are all in metre.

Example 14. *In Young's double experiment, the two parallel slits are made one millimetre apart and a screen is placed one metre away. What is the fringe separation when blue green light of wavelength 500 nm is used ?* [NCERT]

Solution. Here $d = 1 \text{ mm} = 10^{-3}\text{m}$, $D = 1 \text{ m}$

$$\lambda = 500 \text{ nm} = 500 \times 10^{-9}\text{m}$$

Fringe width,

$$\beta = \frac{D\lambda}{d} = \frac{1 \times 500 \times 10^{-9}}{10^{-3}}\text{m}$$
$$= 5 \times 10^{-4} \text{ m} = \mathbf{0.5 \text{ mm}}.$$

Example 15. *Laser light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the bright fringes are separated by 8.1 mm. A second light produces an interference pattern in which the fringes are separated by 7.2 mm. Calculate the wavelength of the second light.* [CBSE D 2000C ; OD 09]

Solution. Here $\lambda_1 = 630$ nm, $\beta_1 = 8.1$ mm,

$$\beta_2 = 7.2 \text{ mm}, \quad \lambda_2 = ?$$

Fringe width, $\beta = \dfrac{D\lambda}{d}$

For constant D and d, $\beta \propto \lambda$

or
$$\frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1}$$

$\therefore$
$$\lambda_2 = \frac{\beta_2}{\beta_1} \times \lambda_1$$
$$= \frac{7.2 \text{ mm}}{8.1 \text{ mm}} \times 630 \text{ nm} = \mathbf{560 \text{ nm}}.$$

Example 16. *Yellow light of wavelength 6000 Å produces fringes of width 0.8 mm in Young's double slit experiment. What will be the fringe width if the light source is replaced by another monochromatic source of wavelength 7500 Å and the separation between the slits is doubled ?* [CBSE Sample Paper 05]

Solution. Here $\lambda_1 = 6000$ Å, $\beta_1 = 0.8$ mm,

$$\lambda_2 = 7500 \text{ Å}$$

Fringe width in first case,

$$\beta_1 = \frac{D\lambda_1}{d}$$

Fringe width in second case,

$$\beta_2 = \frac{D\lambda_2}{2d}$$

$\therefore$
$$\frac{\beta_2}{\beta_1} = \frac{D\lambda_2/2d}{D\lambda_1/d} = \frac{1}{2}\cdot\frac{\lambda_2}{\lambda_1}$$

or
$$\beta_2 = \frac{1}{2}\cdot\frac{\lambda_2}{\lambda_1}\cdot\beta_1$$
$$= \frac{1}{2} \times \frac{7500 \text{ Å}}{6000 \text{ Å}} \times 0.8 \text{ mm} = \mathbf{0.5 \text{ mm}}.$$

Example 17. *The fringe width in a Young's double slit interference pattern is 2.4×10^{-4} m, when red light of wavelength 6400 Å is used. By how much will it change, if blue light of wavelength 4000 Å is used.* **[Haryana 02]**

Solution. Here $\beta_1 = 2.4 \times 10^{-4}$m, $\lambda_1 = 6400$ Å, $\lambda_2 = 4000$ Å

$\therefore$
$$\frac{\beta_2}{\beta_1} = \frac{\lambda_2}{\lambda_1} = \frac{4000}{6400} = \frac{5}{8}$$

or
$$\beta_2 = \frac{5}{8} \times \beta_1$$
$$= \frac{5}{8} \times 2.4 \times 10^{-4} = 1.5 \times 10^{-4} \text{ m}$$

Decrease in fringe width

$$= \beta_1 - \beta_2 = (2.4 - 1.5) \times 10^{-4}$$
$$= \mathbf{0.9 \times 10^{-4} \text{ m}}.$$

Example 18. *In a two slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance D from the slits. If the screen is moved 5×10^{-2} m towards* *the slits, the change in fringe width is 3×10^{-5} m. If the distance between the slits is 10^{-3}m, calculate the wavelength of the light used.* [CBSE D 03, 06C]

Solution. The fringe width in the two cases will be

$$\beta = \frac{D\lambda}{d} \quad \text{and} \quad \beta' = \frac{D'\lambda}{d}$$

$$\therefore \quad \beta - \beta' = \frac{(D - D')\lambda}{d}$$

or wavelength,

$$\lambda = \frac{(\beta - \beta')\, d}{D - D'}$$

But $D - D' = 5 \times 10^{-2}$ m

and $\beta - \beta' = 3 \times 10^{-5}$ m, $d = 10^{-3}$ m

$$\therefore \quad \lambda = \frac{3 \times 10^{-5} \times 10^{-3}}{5 \times 10^{-2}}$$

$$= 6 \times 10^{-7}\text{ m} = \mathbf{6000\ Å}.$$

Example 19. *In Young's double slit experiment, using light of wavelength 400 nm, interference fringes of width 'X' are obtained. The wavelength of light is increased to 600 nm and the separation between the slits is halved. If one wants the observed fringe width on the screen to be the same in the two cases, find the ratio of the distance between the screen and the plane of the interfering sources in the two arrangements.* [CBSE D 04]

Solution. Fringe width X is same in both cases.

In *first case*,

$$X = \frac{D_1 \lambda_1}{d}$$

In *second case*,

$$X = \frac{D_2 \lambda_2}{d/2}$$

$$\therefore \quad \frac{D_1 \lambda_1}{d} = \frac{D_2 \lambda_2}{d/2}$$

or $$\frac{D_1}{D_2} = 2 \cdot \frac{\lambda_2}{\lambda_1} = \frac{2 \times 600}{400} = \frac{3}{1} = \mathbf{3:1}.$$

Example 20. *In Young's experiment, the width of the fringes obtained with light of wavelength 6000 Å is 2.0 mm Calculate the fringe width if the entire apparatus is immersed in a liquid medium of refractive index 1.33.* [CBSE D 03]

Solution. Here $\beta = 2.0$ mm, $\mu = 1.33$

Refractive index of liquid,

$$\mu = \frac{\text{wavelength of light in vacuum}}{\text{wavelength of light in liquid}} = \frac{\lambda}{\lambda'}$$

or $$\lambda' = \frac{\lambda}{\mu}$$

Fringe width in air,

$$\beta = \frac{D\lambda}{d}$$

Fringe width in liquid,

$$\beta' = \frac{D\lambda'}{d} = \frac{D\lambda}{d\mu} = \frac{\beta}{\mu} = \frac{2.0\text{ mm}}{1.33} = \mathbf{1.5\ mm}.$$

Example 21. *In Young's double slit experiment the light has a frequency of 6×10^{14} Hz and distance between the centres of adjacent fringes is 0.75 mm. If the screen is 1.5 m away, what is the distance between the slits ?*

Solution. Here $\nu = 6 \times 10^{14}$ Hz, $c = 3 \times 10^8$ ms^{-1},

$$\beta = 0.75\text{ mm} = 0.75 \times 10^{-3}\text{ m}$$

Wavelength,

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{6 \times 10^{14}} = 5 \times 10^{-7}\text{ m}$$

Distance between the slits,

$$d = \frac{D\lambda}{\beta} = \frac{1.5 \times 5 \times 10^{-7}}{0.75 \times 10^{-3}} = 10^{-3}\text{ m} = \mathbf{1\ mm}.$$

Example 22. *In a Young's double slit experiment, red light of wavelength 6000 Å is used and the nth bright fringe is obtained at a point P on the screen. Keeping the same setting, the source is replaced by green light of 5000 Å and now (n + 1)th bright fringe is obtained at the point P. Calculate the value of n.* [Haryana 02]

Solution. Let x be the distance of point P from the centre of the screen.

When red light ($\lambda = 6000$ Å) is used, nth bright fringe is obtained at point P.

$$\therefore \quad x = \frac{nD\lambda}{d} = \frac{nD \times 6000 \times 10^{-10}}{d}$$

When green light ($\lambda' = 5000$ Å) is used, $(n+1)$th bright fringe is obtained at the same point P.

$$\therefore \quad x = \frac{(n+1)\, D\lambda'}{d} = \frac{(n+1) D \times 5000 \times 10^{-10}}{d}$$

Equating the two values of x, we get

$$\frac{nD \times 6000 \times 10^{-10}}{d} = \frac{(n+1) D \times 5000 \times 10^{-10}}{d}$$

or $$6n = 5(n+1)$$

or $$n = \mathbf{5}.$$

Example 23. *A beam of light consisting of two wavelengths, 800 nm and 600 nm is used to obtain the interference fringes in a Young's double slit experiment on a screen placed 1.4 m away. If the two slits are separated by 0.28 mm, calculate the least distance from the central bright maximum where the bright fringes of the two wavelengths coincide.* [CBSE OD 12]

Solution. The two bright fringes will coincide at the least distance x from the central maximum if

$$x = n\lambda_1 \frac{D}{d} = (n+1)\lambda_2 \frac{D}{d}$$

or $\quad n\lambda_1 = (n+1)\lambda_2$

or $\quad n \times 800 = (n+1) \times 600$

or $\quad 4n = 3n + 3$ or $n = 3$

$$\therefore \quad x = 3\frac{D\lambda_1}{d} = \frac{3 \times 1.4 \times 800 \times 10^{-9}}{0.28 \times 10^{-3}} \text{ m}$$

$$= 12 \times 10^{-3} \text{ m} = \mathbf{12 \text{ mm}}$$

Example 24. *In Young's double slit experiment, the slits are 0.2 mm apart and the screen is 1.5 m away. It is observed that the distance between the central bright fringe and fourth dark fringe is 1.8 cm. Find the wavelength of light used.* [Punjab 92, 93]

Solution. Here $d = 0.2$ mm $= 0.2 \times 10^{-3}$ m, $D = 1.5$ m, $x'_4 = 1.8$ cm $= 1.8 \times 10^{-2}$ m

The distance of nth dark fringe from the central bright fringe is given by

$$x'_n = (2n-1)\frac{D\lambda}{2d},$$

$$\therefore \quad x'_4 = \frac{7}{2} \cdot \frac{D\lambda}{d}$$

or
$$\lambda = \frac{2dx'_4}{7D} = \frac{2 \times 0.2 \times 10^{-3} \times 1.8 \times 10^{-2}}{7 \times 1.5}$$

$$= \mathbf{6.86 \times 10^{-7} \text{ m.}}$$

Example 25. *In a Young's double slit experiment, the slits are separated by 0.5 mm and screen is placed 1.0 m away. It is found that the ninth bright fringe is at a distance of 8.835 mm from the second dark fringe. Find the wavelength of light used.*

Solution. The distance of nth bright fringe from the central bright fringe is

$$x_n = \frac{nD\lambda}{d} = n\beta \quad \therefore \quad x_9 = 9\beta$$

The distance of nth dark fringe from the central bright fringe is

$$x'_n = (2n-1)\frac{D\lambda}{2d} = (2n-1)\frac{\beta}{2}$$

$$\therefore \quad x'_2 = \frac{3}{2}\beta$$

But $\quad x_9 - x'_2 = 8.835$ mm [Given]

or $\quad 9\beta - \frac{3}{2}\beta = 8.835$ mm or $\frac{15}{2}\beta = 8.835$ mm

or $\quad \beta = \frac{8.835 \times 2}{15}$ mm

$$= 1.178 \text{ mm} = 1.178 \times 10^{-3} \text{ m}$$

Hence
$$\lambda = \frac{\beta d}{D} = \frac{1.178 \times 10^{-3} \times 0.5 \times 10^{-3}}{1.0} \text{ m}$$

$$= 0.5890 \times 10^{-6} \text{ m} = \mathbf{5890 \text{ Å}}.$$

Example 26. *In a Young's double experiment, the slits are 1.5 mm apart. When the slits are illuminated by a monochromatic light source and the screen is kept 1 m apart from the slits, width of 10 fringes is measured as 3.93 mm. Calculate the wavelength of light used. What will be the width of 10 fringes when the distance between the slits and the screen is increased by 0.5 m. The source of light used remains the same.* [Karnataka 95]

Solution. *In first case :*

$$d = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}, \; D = 1 \text{ m}$$

Width of 10 fringes $= 3.93$ mm

$\therefore$ Fringe width,

$$\beta = \frac{3.93}{10} = 0.393 \text{ mm} = 0.393 \times 10^{-3} \text{ m}$$

Wavelength,

$$\lambda = \frac{\beta d}{D} = \frac{0.393 \times 10^{-3} \times 1.5 \times 10^{-3}}{1}$$

$$= \mathbf{5.895 \times 10^{-7} \text{ m}}$$

In second case : $D' = 1 + 0.5 = 1.5$ m,

$d = 1.5 \times 10^{-3}$ m, $\lambda = 5.895 \times 10^{-7}$ m

Width of 10 fringes $= 10\beta' = \frac{10 D'\lambda}{d}$

$$= \frac{10 \times 1.5 \times 5.895 \times 10^{-7}}{1.5 \times 10^{-3}} = \mathbf{5.895 \times 10^{-3} \text{ m.}}$$

Example 27. *A double slit is illuminated by light of wavelength 6000 Å. The slits are 0.1 cm apart and the screen is placed 1 m away. Calculate (i) the angular position of 10th maximum in radian and (ii) separation of the adjacent minima.* [Punjab 96]

Solution. (*i*) The angular position of nth maximum is given by

$$\theta_n = \frac{x_n}{D} = \frac{nD\lambda / d}{D} = \frac{n\lambda}{d}$$

$$\therefore \quad \theta_{10} = \frac{10 \times 6000 \times 10^{-10}}{0.1 \times 10^{-2}} = \mathbf{0.006 \text{ rad.}}$$

(*ii*) Separation between two adjacent minima *i.e.*, fringe width,

$$\beta = \frac{D\lambda}{d} = \frac{1 \times 6000 \times 10^{-10}}{0.1 \times 10^{-2}}$$

$$= 6 \times 10^{-4} \text{ m} = \mathbf{0.6 \text{ mm.}}$$

Example 28. *Sodium light has two wavelengths $\lambda_1 = 589$ nm and $\lambda_2 = 589.6$ nm. As the path difference increases, when is the visibility of the fringes minimum ?* [NCERT]

Solution. The visibility of the fringes will be poorest when the path difference p is an integral multiple of λ_1 and a half integral multiple of λ_2. As p is increased, this happens first when

$$\frac{p}{\lambda_1} - \frac{p}{\lambda_2} = \frac{1}{2} \quad \text{or} \quad p\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = \frac{1}{2}$$

or
$$p = \frac{1}{2}\left(\frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\right)$$

Now $\lambda_1 = 589 \text{ nm} = 589 \times 10^{-9} \text{m}$

and $\lambda_2 = 589.6 \text{ nm} = 589.6 \times 10^{-9} \text{m}$

$$\therefore \quad p = \frac{1}{2} \cdot \frac{589 \times 10^{-9} \times 589.6 \times 10^{-9}}{(589.6 - 589) \times 10^{-9}}$$

$$= \frac{1}{2} \cdot \frac{347274.4 \times 10^{-9}}{0.6} \text{ m}$$

$$= 289395.31 \times 10^{-6} \text{mm} = \mathbf{0.29 \text{ mm}}.$$

Problems For Practice

1. In Young's double slit experiment the slits are separated by 0.24 mm. The screen is 1.2 m away from the slits. The fringe width is 0.3 cm. Calculate the wavelength of the light used in the experiment. **[CBSE D 93C]** (Ans. 6000 Å)

2. In Young's double slit experiment, while using a source of light of wavelength 4500 Å, the fringe width obtained is 0.4 cm. If the distance between the slits and the screen is reduced to half, calculate the new fringe width. **[CBSE D 96C]** (Ans. 0.2 cm)

3. In a Young's double slit experiment, interference fringes were produced on a screen placed at 1.5 m from the two slits 0.3 mm apart and illuminated by light of wavelength 6400 Å. Find the fringe width. **[Haryana 02]** (Ans. 3.2 mm)

4. Green light of wavelength 5100 Å from a narrow slit is incident on a double slit. If the overall separation of 10 fringes on a screen 200 cm away is 2 cm, find slit separation. **[Punjab 02]** (Ans. 0.51 mm)

5. In Young's double slit experiment the fringe width obtained is 0.6 cm, when light of wavelength 4800 Å is used. If the distance between the screen and the slit is reduced to half, what should be the wavelength of light used to obtain fringes 0.0045 m wide ? **[Punjab 97]** (Ans. 7.2×10^{-6} m)

6. In Young's double slit experiment, the width of fringes obtained from a source of light of wavelength 5000 Å is 3.6 mm. Calculate the fringe width if the apparatus is immersed in a liquid of refractive index 1.2. **[CBSE OD 96C]** (Ans. 3.0 mm)

7. The two slits in Young's double slit experiment are separated by a distance of 0.03 mm. An interference pattern is produced on a screen 1.5 m away. The 4th bright fringe is at a distance of 1 cm from the central maximum. Calculate the wavelength of light used. **[CBSE OD 95]** (Ans. 5000 Å)

8. In Young's double slit experiment, the two slits 0.15 mm apart are illuminated by monochromatic light of wavelength 450 nm. The screen is 1.0 m away from the slits. Find the distance of the second (*i*) bright fringe, (*ii*) dark fringe from the maximum. **[CBSE OD 10]** [Ans. (*i*) 6 mm (*ii*) 4.5 mm]

9. In Young's double slit experiment, red light of wavelength 620 nm is used and the two slits are 0.3 mm apart. Interference fringes observed on a screen are found to be 1.3 mm apart. Calculate (*i*) the distance of slits from the screen and (*ii*) the fringe width if this distance is doubled. **[Karnataka 92]** [Ans. (*i*) 0.629 m (*ii*) 2.6 mm]

10. When two narrow slits separated by a small distance are illuminated by a light of wavelength 5×10^{-7} m, interference fringes of width 0.5 mm are obtained on a screen. What should be the wavelength of light source to obtain fringes 0.3 mm wide, if the distance between the screen and the slits is reduced to half of the initial value. **[Karnatka 93]** (Ans. 6×10^{-7} m)

11. In a Young's double slit experiment, the distance between the slits and the screen is 1.60 m. Using light of wavelength 6×10^{-7} m, the distance between the centre of the interference pattern and fourth bright fringe on either side is 16 mm. Calculate the slit separation. **[CBSE F 95]** (Ans. 0.24 mm)

12. In Young's double slit experiment, light of wavelength 6000 Å is used to get an interference pattern on a screen. The fringe width changes by 1.5 mm, when the screen is brought towards the double slit by 50 cm. Find the distance between the two slits. **[Haryana 02]** (Ans. 0.2 mm)

13. In Young's experiment, two coherent sources are 1.5 mm apart and fringes are obtained at a distance of 2.5 m from them. If the sources produce light of wavelength 589.3 nm, find the number of fringes in the interference pattern, which is 4.9×10^{-3} m wide. (Ans. 5)

14. In a Young's double slit experiment, the interference fringes are obtained on a screen 0.75 m apart. The third dark band is at a distance of 5.5 mm from the central fringe. (*i*) Determine the wavelength of light used if the two slits are 0.15 mm apart. (*ii*) What will be the wavelength of light used if the entire apparatus is immersed in a liquid of refractive index 4/3? [Ans. (*i*) 4.4×10^{-7} m (*ii*) 3.3×10^{-7} m]

15. In Young's experiment, interference pattern is obtained on a screen at a distance of 1 m from slits separated by 0.05 cm and illuminated by sodium light of wavelength 5893 Å. Calculate distance between 4th bright fringe on one side and 3rd bright fringe on other side of central fringe. (Ans. 8.25 mm)

16. Among two interfering sources, let A be ahead in phase by 54° relative to B. If the observation be taken from point P, such that $PB - PA = 1.5\,\lambda$, deduce the phase difference between the waves from A and B reaching P. (Ans. $3.3\,\pi$)

17. In Young's experiment, what will be the phase difference and the path difference between the light waves reaching (*i*) third bright fringe and (*ii*) third dark fringe from the central fringe. Take λ = 5000 Å.

[Ans. (*i*) $6\,\pi$; 15000 Å (*ii*) $5\,\pi$, 12500 Å]

18. In a Young's double slit interference pattern at a point, we observe the 10th bright fringe (order maxima) for wavelength 7000 Å. What order maxima will be visible if the source of light is replaced by light of wavelength 5000 Å ? (Ans. 14)

19. In Young's double slit experiment, light waves of $\lambda = 5.4 \times 10^2$ nm and $\lambda = 6.85 \times 10^1$ nm are used in turn, keeping the same geometry of the set up. Calculate the ratio of the fringe widths in the two cases. (Ans. 7.9)

20. In a Young's double slit experiment, the two slits are 2 mm apart and the screen is positioned 140 cm away from the plane of the slits. The slits are illuminated with light of wavelength 600 nm. Find the distance of the third bright fringe, from the central maximum, in the interference pattern obtained on the screen.

If the wavelength of the incident light were changed to 480 nm, find the shift in the position of third bright fringe from the central maximum. **[CBSE OD 08C]**

(Ans. 1.26 mm, 0.25 mm)

21. In Young's double slit experiment, two slits are separated by 3 mm distance and illuminated by light of wavelength 480 nm. The screen is 2 m from the plane of the slits. Calculate the separation between the 8th bright fringe and the 3rd dark fringe observed with respect to the central bright fringe. **[CBSE OD 01]** (Ans. 1.76 mm)

HINTS

1. Wavelength, $\lambda = \frac{\beta d}{D} = \frac{0.3 \times 10^{-2} \times 0.24 \times 10^{-3}}{1.2}$ m

$= 0.06 \times 10^{-5}$ m $=$ **6000 Å**.

2. Fringe width in first case, $\beta_1 = \frac{D\lambda}{d}$

Fringe width in second case,

$$\beta_2 = \frac{D}{2}\frac{\lambda}{d} = \frac{1}{2}\beta_1 = \frac{1}{2} \times 0.4 = \mathbf{0.2\ cm.}$$

3. $\beta = \frac{D\lambda}{d} = \frac{1.5 \times 6400 \times 10^{-10}}{0.3 \times 10^{-3}}$ m

$= 3.2 \times 10^{-3}$ m $=$ **3.2 mm.**

4. Here $10\,\beta = 2$ cm.

5. ***In first case :*** $\beta = 0.6$ cm $= 0.6 \times 10^{-2}$ m,

$\lambda = 4800$ Å $= 48 \times 10^{-8}$ m

As $\beta = \frac{D\lambda}{d}$

$\therefore \quad 0.6 \times 10^{-2} = \frac{D \times 48 \times 10^{-8}}{d}$

or $\frac{D}{d} = \frac{0.6 \times 10^{-2}}{48 \times 10^{-8}} = 12500$

In second case : $D' = D/2$, $\beta' = 0.0045$ m

$\therefore \quad \lambda = \frac{\beta' d}{D'} = \frac{2\beta' d}{D} = \frac{2 \times 0.0045}{12500}$

$= \mathbf{7.2 \times 10^{-6}\ m.}$

6. Proceed as in Example 20 on page 10.16.

7. Proceed as in Exercise 10.4 on page 10.82.

8. (*i*) Distance of the second bright fringe from the central maximum,

$$x_2 = \frac{2D\lambda}{d} = \frac{2 \times 1.0 \times 450 \times 10^{-9}}{0.15 \times 10^{-3}} \text{ m}$$

$= 6 \times 10^{-3}$ m $=$ **6 mm.**

(*ii*) Distance of the second dark fringe from the central maximum,

$$x_2' = (2 \times 2 - 1)\frac{D\lambda}{2d} = \frac{3}{2}\frac{D\lambda}{d} = \frac{3}{2} \times 3 \text{ mm}$$

$=$ **4.5 mm.**

9. (*i*) $D = \frac{\beta d}{\lambda} = \frac{1.3 \times 10^{-3} \times 0.3 \times 10^{-3}}{620 \times 10^{-9}} = \mathbf{0.629\ m.}$

(*ii*) $\beta' = \frac{D'\lambda}{d} = \frac{2D\lambda}{d} = \frac{2 \times 0.629 \times 620 \times 10^{-9}}{0.3 \times 10^{-3}}$

$= 2.6 \times 10^{-3}$ m $=$ **2.6 mm.**

10. Here $\lambda = 5 \times 10^{-7}$ m, $\beta = 0.5$ mm $= 0.5 \times 10^{-3}$ m

As $\beta = \frac{D\lambda}{d} \quad \therefore \quad \frac{d}{D} = \frac{\lambda}{\beta} = \frac{5 \times 10^{-7}}{0.5 \times 10^{-3}} = 10^{-3}$

On reducing the distance between the screen and the slits to half of its initial value,

$$\beta' = 0.3 \text{ mm} = 0.3\times10^{-3} \text{ m}, \; D' = \frac{D}{2} \quad \text{and} \quad d' = d$$

$$\therefore \; \lambda' = \beta' \frac{d'}{D'} = 0.3\times10^{-3} \times \frac{d}{D/2} = 0.6\times10^{-3} \frac{d}{D}$$

$$= 0.6\times10^{-3}\times10^{-3} = \mathbf{6\times10^{-7} \text{ m}}.$$

11. $x_4 = \frac{4D\lambda}{d}$

$$d = \frac{4D\lambda}{x_4} = \frac{4\times1.60\times6\times10^{-7}}{16\times10^{-3}}$$

$$= 2.4\times10^{-4}\text{m} = \mathbf{0.24 \text{ mm}}.$$

12. Fringe width, $\beta = \frac{D\lambda}{d}$

As λ and d are constant, so the change in fringe width is given by

$$\Delta\beta = \frac{\lambda\,\Delta D}{d}$$

Distance between the two slits,

$$d = \frac{\lambda\,\Delta D}{\Delta\beta} = \frac{6000\times10^{-10}\times50\times10^{-2}}{1.5\times10^{-3}}$$

$$= 0.2\times10^{-3}\text{m} = \mathbf{0.2 \text{ mm}}.$$

13. Fringe width,

$$\beta = \frac{D\lambda}{d} = \frac{2.5\times589.3\times10^{-9}}{1.5\times10^{-3}} = 982.17\times10^{-6}\text{m}$$

$$\text{Number of fringes} = \frac{\Delta x}{\beta} = \frac{4.9\times10^{-3}}{982.17\times10^{-6}} \simeq \mathbf{5}.$$

14. (*i*) $x'_3 = (2\times3-1)\frac{D\lambda}{2d}$

or $5.5\times10^{-3} = \frac{5\times0.75\times\lambda}{2\times0.15\times10^{-3}}$

$\therefore \quad \lambda = \mathbf{4.4\times10^{-7} \text{ m}}.$

(*ii*) Wavelength in liquid,

$$\lambda' = \frac{\lambda}{\mu} = \frac{4.4\times10^{-7}}{4/3} = \mathbf{3.3\times10^{-7} \text{ m}}.$$

15. Required distance

$$= x_4 + x_3 = \frac{4D\lambda}{d} + \frac{3D\lambda}{d} = \frac{7D\lambda}{d}$$

$$= \frac{7\times1\times5893\times10^{-10}}{0.05\times10^{-2}}$$

$$= 8.25\times10^{-3}\text{m} = \mathbf{8.25 \text{ mm}}.$$

16. $\phi_1 = 54° = 54\times\frac{\pi}{180} = 0.3\,\pi$ rad ;

$$\phi_2 = \frac{2\pi\,p}{\lambda} = \frac{2\pi}{\lambda}\times1.5\,\lambda = 3\pi \text{ rad}$$

Total phase difference, $\phi = \phi_1 + \phi_2 = \mathbf{3.3\pi \text{ rad}}.$

17. For third bright fringe, $p = 3\lambda$ and

$$\phi = 3\times2\pi = 6\pi \text{ rad}$$

For third dark fringe,

$$p = (2n-1)\frac{\lambda}{2} = (2\times3-1)\frac{\lambda}{2} = \frac{5\lambda}{2}$$

and $\phi = \frac{5}{2}\times2\pi = \mathbf{5\pi \text{ rad}}.$

18. As $n_1\lambda_1 = n_2\lambda_2 \;\therefore\; n_2 = \frac{\lambda_1}{\lambda_2}\cdot n_1 = \frac{7000}{5000}\times10 = \mathbf{14}.$

19. Ratio of fringe widths, $\frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2} = \frac{5.4\times10^2}{6.85\times10^1} = \mathbf{7.9}.$

20. Here $d = 2$ mm $= 2\times10^{-3}$m, $D = 140$ cm $= 1.40$ m

When $\lambda = 600$ nm,

$$x_3 = \frac{3D\lambda}{d} = \frac{3\times1.40\times600\times10^{-9}}{2\times10^{-3}}$$

$$= 1.26\times10^{-3}\text{m} = \mathbf{1.26 \text{ mm}}.$$

When $\lambda = 480$ nm,

$$x_3 = \frac{3\times1.40\times480\times10^{-9}}{2\times10^{-3}}$$

$$= 1.01\times10^{-3}\text{m} = \mathbf{1.01 \text{ mm}}$$

Shift in the position of third bright fringe

$$= 1.26 - 1.01 = \mathbf{0.25 \text{ mm}}.$$

21. Required distance

$$= x_8 - x'_3 = \frac{8D\lambda}{d} - \frac{5D\lambda}{2d} = \frac{11D\lambda}{2d}$$

$$= \frac{11\times2\times480\times10^{-9}}{2\times3\times10^{-3}}$$

$$= 1.76\times10^{-3}\text{m} = \mathbf{1.76 \text{ mm}}.$$

10.13 CONDITIONS FOR SUSTAINED INTERFERENCE

14. *What is a sustained interference pattern ? State the necessary conditions for obtaining a sustained interference of light.*

Sustained interference pattern. In order to observe an interference pattern, it is necessary that the positions of maxima and minima do not keep on changing with time, otherwise the maxima and minima of intensity will mix up to produce uniform illumination. *The interference pattern, in which the positions of maxima and minima of intensity on the observation screen do not change with time, is called a* ***sustained*** *or* ***permanent interference pattern.***

Conditions for sustained interference. The necessary conditions for obtaining a sustained and observable interference pattern of light are as follows :

1. The two sources should continuously emit waves of same frequency or wavelength.

2. The two sources of light should be *coherent, i.e.,* they must vibrate either in the same phase or with a constant phase difference between them.
3. For a better contrast between maxima and minima of intensity, the amplitudes of the interfering waves should be equal.
4. The two sources should be narrow, otherwise interference will occur between waves of different parts of the same source and contrast will be poor.
5. The interfering waves must travel nearly along the same direction.
6. The sources should be *monochromatic,* otherwise fringes of different colours will overlap just to give a few observable fringes.
7. The interfering waves should be in the same state of polarisation.
8. To have sufficient fringe width, the distance between the two coherent sources should be small and the distance between the two sources and the screen should be large.

10.14 INTENSITY DISTRIBUTION CURVE FOR INTERFERENCE

15. *Draw and explain the graph showing the variation of intensity in the interference pattern in Young's double slit experiment.*

Intensity distribution curve for interference. Suppose the two interfering waves have the same amplitude a.

The intensity of a bright fringe will be

$$I_{max} = k\,(a + a)^2 = 4ka^2 = \text{constant}$$

So all bright fringes will have the same maximum intensity.

The intensity of a dark fringe will be

$$I_{min} = k\,(a - a)^2 = 0$$

So all dark fringes will be perfectly dark.

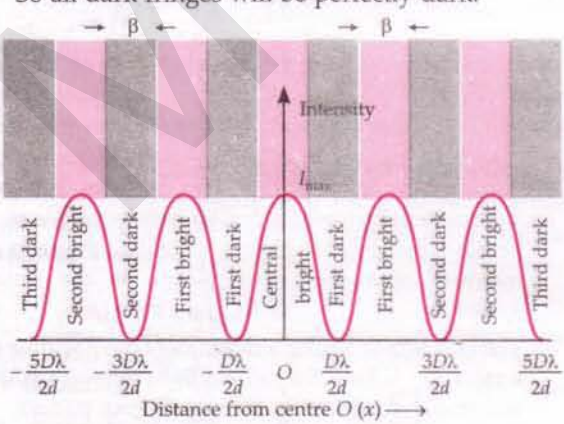


Fig. 10.15 Intensity distribution curve.

On plotting the intensities of bright and dark fringes against distance x from O, we get a curve as shown in Fig. 10.15. The intensity is maximum at the central point O. Then it becomes zero and maximum alternately on either side of O, depending on x is odd multiple of $\frac{D\lambda}{2d}$ and integral multiple of $\frac{D\lambda}{d}$ respectively.

10.15 CONSERVATION OF ENERGY IN INTERFERENCE

16. *Show that the phenomenon of interference is in accordance with the law of conservation of energy.*

Conservation of energy in interference. In an interference pattern, the intensities at the points of maxima and minima are such that

$$I_{max} \propto (a_1 + a_2)^2 \text{ and } I_{min} \propto (a_1 - a_2)^2$$

$$\therefore \quad I_{av} \propto \frac{(a_1 + a_2)^2 + (a_1 - a_2)^2}{2} \text{ or } I_{av} \propto a_1^2 + a_2^2$$

If there is no interference between the light waves from the two sources, then intensity at every point would be same. That is,

$$I = I_1 + I_2 \propto a_1^2 + a_2^2$$

which is same as I_{av} in the interference pattern. So there is no violation of the law of conservation of energy in interference. Whatever energy disappears from a dark fringe, an equal energy appears in a bright fringe.

10.16 COMPARISON OF INTENSITIES AT MAXIMA AND MINIMA

17. *Derive an expression for the ratio of intensities at maxima and minima in an interference pattern.*

Comparison of intensities at maxima and minima. Let a_1 and a_2 be the amplitudes and I_1 and I_2 be the intensities of light waves from two different sources.

As Intensity $\propto$ Amplitude^2 $\quad \therefore \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$

Amplitude at a maximum in interference pattern

$$= a_1 + a_2$$

Amplitude at a minimum in interference pattern

$$= a_1 - a_2$$

Therefore, the ratio of intensities at maxima and minima is

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{\left(\frac{a_1}{a_2} + 1\right)^2}{\left(\frac{a_1}{a_2} - 1\right)^2} \quad \text{or} \quad \frac{I_{max}}{I_{min}} = \left[\frac{r+1}{r-1}\right]^2$$

where $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}}$ = *amplitude ratio* of the two waves.

NOTE The intensity of light through a slit is proportional to its width. If w_1 and w_2 are the widths of the two slits S_1 and S_2, then

$$\frac{w_1}{w_2}=\frac{I_1}{I_2}=\frac{a_1^2}{a_2^2}=r^2.$$

Examples based on Intensity Ratio at Maxima and Minima of an Interference Pattern

Formulae Used

1. Intensity of light ∝ Width of slit
2. Ratio of slit widths, $\frac{w_1}{w_2}=\frac{I_1}{I_2}=\frac{a_1^2}{a_2^2}$
3. Intensity at maxima, $I_{max} \propto (a_1+a_2)^2$
4. Intensity at minima, $I_{min} \propto (a_1-a_2)^2$
5. Intensity ratio at maxima and minima,

$$\frac{I_{max}}{I_{min}}=\frac{(a_1+a_2)^2}{(a_1-a_2)^2}=\left(\frac{r+1}{r-1}\right)^2$$

where $r=\frac{a_1}{a_2}=\sqrt{\frac{I_1}{I_2}}$ = amplitude ratio of two waves.

Units Used

Ratios I_1/I_2, w_1/w_2 and I_{max}/I_{min} have no units.

Example 29. *What is the ratio of slit widths if the amplitudes of light waves from them have a ratio of $\sqrt{2}:1$?*

Solution. Width ratio,

$$\frac{w_1}{w_2}=\frac{I_1}{I_2}=\frac{a_1^2}{a_2^2}=\left(\frac{\sqrt{2}}{1}\right)^2=\mathbf{2:1}.$$

Example 30. *Two coherent sources have intensities in the ratio 25 : 16. Find the ratio of the intensities of maxima to minima, after interference of light occurs.* [CBSE D 03C]

Solution. Amplitude ratio,

$$r=\sqrt{\frac{I_1}{I_2}}=\sqrt{\frac{25}{16}}=\frac{5}{4}$$

$$\therefore \quad \frac{I_{max}}{I_{min}}=\frac{(r+1)^2}{(r-1)^2}=\frac{\left(\frac{5}{4}+1\right)^2}{\left(\frac{5}{4}-1\right)^2}=81:1.$$

Example 31. *If the two slits in Young's double-slit experiment have width ratio 4 : 1, deduce the ratio of intensity at maxima and minima in the interference pattern.* [CBSE OD 15C]

Solution. Amplitude ratio of the interfering waves,

$$r=\frac{a_1}{a_2}=\sqrt{\frac{I_1}{I_2}}=\sqrt{\frac{w_1}{w_2}}=\sqrt{\frac{4}{1}}=2$$

$$\therefore \quad \frac{I_{max}}{I_{min}}=\left(\frac{r+1}{r-1}\right)^2=\left(\frac{2+1}{2-1}\right)^2=\frac{9}{1}=\mathbf{9:1}.$$

Example 32. *The ratio of the intensities at minima to the maxima in the Young's double slit experiment is 9:25. Find the ratio of the widths of the two slits.* [CBSE OD 14]

Solution. $\frac{I_{min}}{I_{max}}=\frac{9}{25}$ or $\left(\frac{r-1}{r+1}\right)^2=\frac{9}{25}$

or $\frac{r-1}{r+1}=\frac{3}{5}$ or $5r-5=3r+3$

or $r=4=\frac{a_1}{a_2}$, the amplitude ratio

∴ Width ratio of two slits,

$$\frac{w_1}{w_2}=\frac{I_1}{I_2}=\frac{a_1^2}{a_2^2}=\frac{16}{1}=\mathbf{16:1}$$

Example 33. *Two coherent sources of light of intensity ratio β interfere. Prove that in the interference pattern,*

$$\frac{I_{max}-I_{min}}{I_{max}+I_{min}}=\frac{2\sqrt{\beta}}{1+\beta}.$$

Solution. Here $\beta=\frac{I_1}{I_2}=\frac{a_1^2}{a_2^2}$ or $\frac{a_1}{a_2}=\sqrt{\beta}$

As $I_{max}=k(a_1+a_2)^2$ and $I_{min}=k(a_1-a_2)^2$

$$\therefore \quad \frac{I_{max}-I_{min}}{I_{max}+I_{min}}=\frac{k(a_1+a_2)^2-k(a_1-a_2)^2}{k(a_1+a_2)^2+k(a_1-a_2)^2}$$

$$=\frac{(a_1+a_2)^2-(a_1-a_2)^2}{(a_1+a_2)^2+(a_1-a_2)^2}$$

$$=\frac{4a_1a_2}{2(a_1^2+a_2^2)}$$

$$=\frac{2(a_1/a_2)}{\left[\frac{a_1^2}{a_2^2}+1\right]}=\frac{\mathbf{2\sqrt{\beta}}}{\mathbf{\beta+1}}.$$

Problems For Practice

1. Two coherent sources, whose intensity ratio is 16 : 1 produce interference fringes. Calculate the ratio of intensity of maxima and minima in the fringe system. [Haryana 01] (Ans. 25 : 9)
2. The two slits in Young's experiment have widths in a ratio 25 : 1. Find the ratio of light intensity at the maxima and minima in the interference pattern. [AIPMT 15] (Ans. 9 : 4)

3. If the two slits in Young's experiment have width ratio 4 : 9, deduce the ratio of intensity at maxima and minima in interference pattern. **[Himachal 02]** (Ans. 25 : 1)

4. The ratio of intensity at maxima and minima is 25 : 16. What will be the ratio of the width of the two slits in Young's double slit experiment ? **[CBSE F 06]** (Ans. 81 : 1)

5. Two coherent sources whose intensity ratio is 81 : 1 produce interference fringes. Calculate the ratio of intensity of maxima and minima in the fringe system. (Ans. 25 : 16)

6. The width of one of the two slits in a Young's double slit experiment is double of the other slit. Assuming that the amplitude of the light coming from a slit is proportional to the slit width, find the ratio of the maximum to the minimum intensity in the interference pattern. (Ans. 9 : 1)

HINTS

1. $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{16}{1}} = 4$

$$\frac{I_{max}}{I_{min}} = \left(\frac{r+1}{r-1}\right)^2 = \left(\frac{4+1}{4-1}\right)^2 = \mathbf{25 : 9.}$$

2. Amplitude ratio, $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{w_1}{w_2}} = \sqrt{\frac{25}{1}} = 5$

Hence

$$\frac{I_{max}}{I_{min}} = \left(\frac{r+1}{r-1}\right)^2 = \left(\frac{5+1}{5-1}\right)^2 = \left(\frac{6}{4}\right)^2 = \frac{9}{4} = \mathbf{9 : 4.}$$

3. Proceed as in Problem 2 above.

4. Given $\frac{I_{max}}{I_{min}} = \frac{25}{16}$ or $\left[\frac{r+1}{r-1}\right]^2 = \frac{25}{16}$

or $\frac{r+1}{r-1} = \frac{5}{4}$ or $r = 9 = \frac{a_1}{a_2}$

∴ Width ratio of slits

$$\frac{w_1}{w_2} = \frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = (9)^2 = \frac{81}{1} = \mathbf{81 : 1}$$

5. Amplitude ratio, $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{81}{1}} = 9$

The ratio of intensity of maxima and minima is

$$\frac{I_{max}}{I_{min}} = \left[\frac{r+1}{r-1}\right]^2 = \left[\frac{9+1}{9-1}\right]^2 = \frac{100}{64} = \mathbf{25 : 16.}$$

6. Let a and $2a$ be the amplitudes of the two waves coming from the two slits. Then

$$a_{max} = 2a + a = 3a \quad \text{(Constructive interference)}$$

$$a_{min} = 2a - a = a \quad \text{(Destructive interference)}$$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{a_{max}^2}{a_{min}^2} = \frac{(3a)^2}{a^2} = \mathbf{9 : 1.}$$

10.17 INTERFERENCE PATTERN WITH WHITE LIGHT

18. *Explain the effect on the interference fringes in a Young's double slit experiment when the monochromatic source is replaced by a source of white light.*

Interference pattern with white light. White light consists of colours from violet to red with wavelength range from 4000 Å to 7000 Å. Different component colours of white light produce their own interference pattern. At the centre of the screen, the path difference is zero for all such components. So bright fringes of different colours overlap at the centre. Consequently, the central fringe is white.

Now fringe width $\beta = D\lambda / d$ *i.e.*, $\beta \propto \lambda$. Since the violet colour has the lowest λ, the closest fringe on either side of the central while fringe is violet, while the farthest fringe is red. After a few fringes, the interference pattern is lost due to large overlapping of the fringes and uniform white illumination is seen on the screen.

10.18 INTERFERENCE IN THIN FILMS

19. *A light ray suffers multiple reflections and refractions at a thin film of thickness t and refractive index μ. Write expressions for the path difference between any two consecutive rays in the reflected and transmitted systems. Show that the two systems are complimentary of one another. Also explain the formation of colours in thin films.*

Interference in thin films. *A thin film means an extremely small thickness of a transparent medium.* A soap film or a thin film of oil spread over water, when seen in the reflected white light, shows beautiful colours. This is due to the interference between the light waves reflected by the upper and lower surfaces of thin films. As they both originate from the same source, they are coherent waves.

As shown in Fig. 10.16, consider a parallel sided thin film of thickness t and refractive index μ. Suppose a ray SA of monochromatic light is incident on its upper surface. This ray suffers partial reflections and refractions successively at points A, B, C etc ; giving a set of *parallel reflected rays* R_1, R_2, ... and a set of *parallel transmitted rays* T_1, T_2, ... When these rays are focussed by our eyelens, interference patterns are visible.

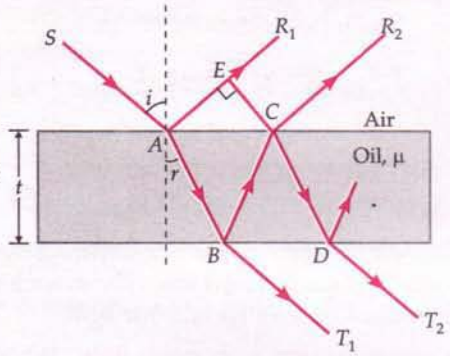


Fig. 10.16 Interference in thin films.

Interference in reflected light. Draw CE perpendicular to AR_1. Then the path difference between two successive reflected rays R_1 and R_2 is

$$p = (AB + BC) \text{ in thin film} - AE \text{ in air}$$
$$= \mu\,(AB + BC) \text{ in air} - AE \text{ in air}$$

or $$p = 2\,\mu t \cos r$$

[From the geometry of the figure]

where r is the angle of refraction. As the ray R_1 is reflected by the upper surface of thin film (denser medium), it suffers an extra path difference of $\lambda/2$.

$\therefore$ Net path difference $= 2\mu t \cos r + \dfrac{\lambda}{2}$

For a bright fringe :

$$2\,\mu t \cos r + \frac{\lambda}{2} = n\,\lambda$$

or $$2\,\mu t \cos r = (2n - 1)\frac{\lambda}{2}$$

or $$2\,\mu t \cos r = (2n + 1)\frac{\lambda}{2}, \quad n = 0, 1, 2, 3, ...$$

For a dark fringe :

$$2\,\mu t \cos r + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$$

or $$2\mu t \cos r = n\,\lambda, \quad n = 0, 1, 2, 3, ...$$

Interference in transmitted light. As the transmitted rays do not suffer any reflection from the surface of a denser medium, the path difference between any two successive rays will be

$$p = 2\,\mu t \cos r$$

$\therefore$ *For a bright fringe,*

$$2\,\mu t \cos r = n\lambda$$

For a dark fringe,

$$2\mu t \cos r = (2n + 1)\frac{\lambda}{2}$$

Obviously, the conditions for maxima and minima in the reflected system are just opposite to those for the transmitted system. Thus the reflected and transmitted systems are *complimentary, i.e.,* a film which appears bright by reflected light, will appear dark by transmitted light and *vice versa.*

Colours in thin films. When a thin film is seen with monochromatic light, we find alternate bright and dark fringes. But with white light, brilliant colours are seen. This is because the path difference $2\,\mu t \cos r$ between any two successive rays depends on μ, t and r. For a particular part of the thin film and for a particular position of the eye, t and r are fixed. But μ varies with the wavelength of light. Different constituents of white light have different wavelengths (λ varying from 4000 Å to 7500 Å), so the conditions for maxima and minima for different constituents occur at different points of thin film. *For example,* if at some place $2\,\mu t \cos r$ equals $1\,\lambda$ for red (7500 Å), it will be $1.5\,\lambda$ for blue (5000 Å), so that at this place blue colour is best reflected, red colour is not reflected at all and the intermediate colours have intermediate contributions. Clearly, at each place there is a mixture of colours, and the composition of this mixture is different at different places. As a result, the reflected light shows various beautiful shades.

Examples based on
Interference in Thin Film

Formulae Used

1. For reflected system of light,
   (*i*) *Maxima* : $2\,\mu t \cos r = (2n + 1)\dfrac{\lambda}{2}$
   (*ii*) *Minima* : $2\,\mu t \cos r = n\lambda$
2. For transmitted system of light,
   (*i*) *Maxima* : $2\,\mu t \cos r = n\lambda$
   (*ii*) *Minima* : $2\,\mu t \cos r = (2n + 1)\dfrac{\lambda}{2}$

   where $n = 0, 1, 2, 3, ...$

Units Used

Thickness t and wavelength λ are in metre, μ and n have no units.

Example 34. *White light may be considered to have λ from 4000 Å to 7500 Å. If an oil film has thickness 10^{-6} m, deduce the wavelengths in the visible region for which the reflection along the normal direction will be (i) weak, (ii) strong. Take μ of the oil as 1.40.*

Solution. The condition for dark fringe or weak reflection when seen in reflected light is $2\,\mu t \cos r = n\lambda$, where n is an integer.

For normal incidence, $r=0$ and $\cos r=1$

so that $2\mu t=n\lambda$ or $\lambda=2\mu t/n$

Substituting the values of μ and t, we get

$$\lambda=\frac{2\times1.4\times10^{-6}}{n}=\frac{28\times10^{-7}}{n}\text{ m}$$

For values of $n<4$ or >7, the values of λ do not lie in the visible range 4000 Å to 7500 Å. But for values of $n=4,5,6,7$, the following wavelengths lie in the visible region :

(i) $\lambda=\frac{28\times10^{-7}}{4}=7.0\times10^{-7}\text{ m}=\mathbf{7000\ Å}$.

(ii) $\lambda=\frac{28\times10^{-7}}{5}=5.6\times10^{-7}\text{ m}=\mathbf{5600\ Å}$.

(iii) $\lambda=\frac{28\times10^{-7}}{6}=4.667\times10^{-7}\text{ m}=\mathbf{4667\ Å}$.

(iv) $\lambda=\frac{28\times10^{-7}}{7}=4.0\times10^{-7}\text{ m}=\mathbf{4000\ Å}$.

The condition for bright fringe or strong reflection is

$$2\mu t=\frac{(2n+1)\lambda}{2}\quad\text{or}\quad\lambda=\frac{4\mu t}{(2n+1)}$$

Substituting the values of μ and t, we get

$$\lambda=\frac{4\times1.4\times10^{-6}}{2n+1}=\frac{56\times10^{-7}}{2n+1}\text{ m}$$

For values of $n<4$ or >6, the values of λ do not lie in the visible range. But for $n=4,5,6$ the following wavelengths lie in the visible range :

(i) $\lambda=\frac{56\times10^{-7}}{2\times4+1}=6.222\times10^{-7}\text{ m}=\mathbf{6222\ Å}$.

(ii) $\lambda=\frac{56\times10^{-7}}{2\times5+1}=5.091\times10^{-7}\text{ m}=\mathbf{5091\ Å}$.

(iii) $\lambda=\frac{56\times10^{-7}}{2\times6+1}=4.308\times10^{-7}\text{ m}=\mathbf{4308\ Å}$.

Example 35. *In a certain region of a wedge-shaped film, 10 fringes are observed with a light source of wavelength 4358 Å. If the wavelength of the light source is changed to 5893 Å, then how many fringes will be observed in the same region of the film ?*

Solution. Let n_1 and n_2 be the number of fringes observed between the points A and B corresponding to wavelengths λ_1 and λ_2. If the thickness of the film changes by Δt between A and B, then for normal incidence

$$2\mu\,\Delta t=n_1\lambda_1=n_2\lambda_2$$

$$\therefore\quad n_2=\frac{n_1\lambda_1}{\lambda_2}$$

But $n_1=10$, $\lambda_1=4358$ Å, $\lambda_2=5893$ Å

$$\therefore\quad n_2=\frac{10\times4358}{5893}=\mathbf{7.4\ fringes.}$$

Example 36. *For light of wavelength $\lambda=6.0\times10^{-7}$ m, it is found that in a thin film of air, 9 fringes occur between two points. Deduce the difference of film thickness between these points.*

Solution. The conditions for two maxima at different thicknesses of the thin film (for normal incidence) may be written as

$$2\mu t_1=(2n_1+1)\frac{\lambda}{2}\quad\text{and}\quad 2\mu t_2=(2n_2+1)\frac{\lambda}{2}$$

$$\therefore 2\mu(t_2-t_1)=(n_2-n_1)\lambda$$

Here $\lambda=6.0\times10^{-7}$, $(n_2-n_1)=9$ and for air, $\mu\simeq1$.

$\therefore$ Difference of film thickness

$$\Delta t=t_2-t_1=\frac{(n_2-n_1)\lambda}{2\mu}=\frac{9\times6.0\times10^{-7}}{2\times1}\text{ m}$$

$$=2.7\times10^{-6}\text{ m}=\mathbf{2.7\ microns.}$$

Example 37. *A parallel beam of sodium light of wavelength 5890 Å is incident on a thin glass plate of refractive index 1.5 such that the angle of refraction in the plate is 60°. Calculate the smallest thickness of the plate which will make it dark by reflection.*

Solution. Here $\lambda=5890\text{ Å}=5890\times10^{-10}\text{ m}$,

$$\mu=1.5,\quad r=60°$$

The condition for minimum thickness corresponding to a dark band is

$$2\mu t\cos r=\lambda$$

$\therefore$ Required thickness,

$$t=\frac{\lambda}{2\mu\cos r}=\frac{5890\times10^{-10}}{2\times1.5\times\cos60°}\text{ m}$$

$$=3928.7\times10^{-10}\text{ m}=\mathbf{3928\ Å.}$$

Example 38. *A soap film is illuminated by white light incident at an angle of 30°. The reflected light is examined by a spectroscope in which a dark band corresponding to wavelength 6000 Å is found. Calculate the minimum thickness of the film. Given refractive index of film $=\frac{4}{3}$*

Solution. Here $i=30°$, $\lambda=6000\text{ Å}=6\times10^{-7}\text{ m}$,

$$\mu=\frac{4}{3}$$

From Snell's law,

$$\sin r=\frac{\sin i}{\mu}=\frac{1/2}{4/3}=\frac{3}{8}$$

$$\therefore\quad\cos r=\sqrt{1-\sin^2 r}=\sqrt{1-\frac{9}{64}}=\sqrt{\frac{55}{64}}=0.927$$

The condition for minimum thickness corresponding to a dark band is $2\mu t \cos r = \lambda$

$\therefore$ Required thickness,

$$t = \frac{\lambda}{2\mu \cos r} = \frac{6 \times 10^{-7}}{2 \times \frac{4}{3} \times 0.927} \text{ m}$$

$$= \mathbf{2.42 \times 10^{-7} \text{ m.}}$$

Example 39. *White light reflected at perpendicular incidence from a soap film has, in the visible spectrum, an interference maximum at 6000 Å and a minimum at 4500 Å with no minimum in between. If $\mu = 4/3$ for the film, what is the film thickness ?*

Solution. Here $\lambda_1 = 6000 \text{ Å} = 6 \times 10^{-7} \text{m}$,

$\lambda_2 = 4500 \text{ Å} = 4.5 \times 10^{-7} \text{m}$, $\mu = \frac{4}{3}$

For normal incidence, the condition for $(n+1)$th maximum is

$$2\mu t = \left(n + \frac{1}{2}\right)\lambda_1 \text{ or } n + \frac{1}{2} = \frac{2\mu t}{\lambda_1} \quad ...(i)$$

The condition for $(n+1)$th minimum is

$$2\mu t = (n+1)\lambda_2 \quad \text{or} \quad n + 1 = \frac{2\mu t}{\lambda_2} \quad ...(ii)$$

Subtracting (*i*) from (*ii*), we get

$$\frac{1}{2} = 2\mu t\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right] = 2\mu t\left[\frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right]$$

$$\therefore \quad t = \frac{\lambda_1\lambda_2}{4\mu(\lambda_1 - \lambda_2)} = \frac{6 \times 10^{-7} \times 4.5 \times 10^{-7}}{4 \times \frac{4}{3} \times (6 - 4.5)10^{-7}}$$

$$= \frac{27}{8} \times 10^{-5} \text{m} = \mathbf{3.375 \times 10^{-5} \text{m.}}$$

Example 40. *A soap film of $\mu = \frac{4}{3}$ is illuminated by white light incident at an angle of 45°. The transmitted light is examined by spectroscope and bright fringe is found to be for wavelength of 6000 Å. Find the minimum thickness of the film.*

Solution. Here $\mu = \frac{4}{3}$, $i = 45°$,

$$\lambda = 6000 \text{ Å} = 6 \times 10^{-7} \text{m}$$

As $\quad \mu = \frac{\sin i}{\sin r}$

$$\therefore \quad \sin r = \frac{\sin i}{\mu} = \frac{\sin 45°}{4/3} = \frac{3}{4\sqrt{2}}$$

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{9}{32}} = \sqrt{\frac{23}{32}} = 0.8478$$

For a bright fringe in transmitted light,

$$2\mu t \cos r = n\lambda$$

For minimum thickness, $n = 1$

$$\therefore \quad 2\mu t \cos r = \lambda$$

or $$t = \frac{\lambda}{2\mu \cos r} = \frac{6 \times 10^{-7}}{2 \times \frac{4}{3} \times 0.8478} = \mathbf{2.6 \times 10^{-7} \text{m.}}$$

Example 41. *White light is incident on a soap film at an angle of $\sin^{-1}\frac{4}{5}$ and the reflected light on examination by the spectroscope shows dark bands. The consecutive dark bands correspond to wavelengths 6100 Å and 6000 Å. If the refractive index of the film is 4/3, calculate its thickness.*

Solution. Here $i = \sin^{-1}\frac{4}{5} \quad \therefore \quad \sin i = \frac{4}{5}$

As $\quad \mu = \frac{\sin i}{\sin r} \quad \therefore \quad \sin r = \frac{\sin i}{\mu} = \frac{4/5}{4/3} = \frac{3}{5} = 0.6$

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - (0.6)^2} = 0.8$$

For a dark fringe in the reflected light,

$$2\mu t \cos r = n\lambda \quad n = 0, 1, 2, 3, ...$$

Suppose *n*th and $(n+1)$th dark bands correspond to wavelengths 6100 Å and 6000 Å respectively. Then

In *first case*,

$$2 \times \frac{4}{3} \times t \times 0.8 = n \times 6100 \times 10^{-10} \quad ...(i)$$

In *second case*,

$$2 \times \frac{4}{3} \times t \times 0.8 = (n+1) \times 6000 \times 10^{-10}$$

$$\therefore \quad n \times 6100 \times 10^{-10} = (n+1) \times 6000 \times 10^{-10}$$

or $$n = 60$$

Putting the value of *n* in equation (*i*), we get

$$2 \times \frac{4}{3} \times t \times 0.8 = 60 \times 6100 \times 10^{-10}$$

or $$t = \mathbf{1.716 \times 10^{-5} \text{m.}}$$

Problems For Practice

1. A soap film 5×10^{-5} cm thick is viewed at an angle of 35° to the normal. Find the wavelength of light in the visible spectrum which will be absent from reflected light ($\mu = 1.33$). Given wavelength range of the visible spectrum is 3900 Å to 7800 Å.

   (Ans. 6000 Å, 4000 Å)

2. A thin film of thickness 4×10^{-5} cm and $\mu = 1.5$ is illuminated by white light incident normal to its surface. What wavelength in the visible range be intensified in the reflected beam ? (Ans. 4800 Å)

3. In a thin film, between two points A and B, eight fringes are observed with light of wavelength 5461 Å. How many fringes will be observed between the same two points A and B if the wavelength of light used is 6500 Å ? (Ans. 6.72 fringes)
4. With a thin air film between two points, 6 fringes appear when light of wavelength 5890 Å is used. Calculate the difference in the thickness of the film between the two points. (Ans. 1.767 micron)
5. White light is incident normally on a plane parallel thin film of $\mu = 1.5$. Find the minimum thickness of the film for which light of $\lambda = 4000$ Å is absent from the reflected light. (Ans. 1.33×10^{-7} m)
6. A soap film of refractive index $4/3$ and thickness 1.5×10^{-4} cm is illuminated by white light incident at angle of 45°. The reflected light is examined by a spectroscope in which a dark band corresponding to the wavelength 5×10^{-5} cm is found. Find the order of the interference band. (Ans. $n = 7$)
7. White light is incident on a soap film of $\mu = 4/3$ at an angle of 30°. On examining the transmitted light with a spectrometer, a dark band of wavelength 5.5×10^{-7} m is found. Find the minimum thickness of the film. (Ans. 1.11×10^{-7} m)

HINTS

1. Here $i = 35°$, $\mu = 1.33$, $t = 5 \times 10^{-7}$ m.

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 35°}{1.33} = \frac{0.5736}{1.33} = 0.4313$$

$\therefore r = 25°33'$ and $\cos r = 0.9022$

For a dark fringe in reflected system,

$$2\mu t \cos r = n\lambda$$

$$\therefore \quad \lambda = \frac{2\mu t \cos r}{n} = \frac{2 \times 1.33 \times 5 \times 10^{-7} \times 0.9022}{n}$$

$$= \frac{1.2 \times 10^{-6}}{n}\,\text{m} = \frac{12000}{n}\,\text{Å}$$

(*i*) For $n = 2$, $\lambda =$ **6000 Å.**

(*ii*) For $n = 3$, $\lambda =$ **4000 Å.**

2. Here $r = 0°$, $\cos r = 1$, $t = 4 \times 10^{-7}$ m, $\mu = 1.5$

For maximum intensity in the reflected system,

$$2\mu t \cos r = \left(n + \frac{1}{2}\right)\lambda$$

$$\therefore \quad \lambda = \frac{2\mu t \cos r}{n + \frac{1}{2}} = \frac{2 \times 1.5 \times 4 \times 10^{-7} \times 1}{n + \frac{1}{2}}$$

$$= \frac{12 \times 10^{-7}}{n + \frac{1}{2}}\,\text{m} = \frac{12000}{n + \frac{1}{2}}\,\text{Å}$$

For $n = 2$, $\lambda = \frac{12000}{5/2} =$ **4800 Å.**

3. Proceed as in Example 35 on page 10.25.
4. Proceed as in Example 37 on page 10.25.
5. Use $2\mu t = \lambda$.
6. Proceed as in Example 38 and use $2\mu t \cos r = n\lambda$.
7. Here $\mu = \frac{4}{3}$, $i = 30°$, $\lambda = 5.5 \times 10^{-7}$ m,

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 30°}{4/3} = \frac{3}{8}$$

$$\therefore \quad \cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{9}{64}}$$

$$= \frac{\sqrt{55}}{8} = \frac{7.4162}{8} = 0.927$$

For a dark fringe in the transmitted system,

$$2\mu t \cos r = (2n + 1)\frac{\lambda}{2}$$

For minimum thickness, $n = 0$

$$\therefore \quad t = \frac{\lambda}{4\mu \cos r} = \frac{5.5 \times 10^{-7}}{4 \times \frac{4}{3} \times 0.927}$$

$$= \mathbf{1.11 \times 10^{-7}\ m.}$$

10.19 DISPLACEMENT OF INTERFERENCE FRINGES

20. *Write an expression for the lateral displacement of the fringes when a thin transparent sheet is inserted in the path of one of the interfering beams.*

Displacement of interference fringes. When a thin transparent sheet of thickness t and refractive index μ is inserted in the path of one of the interfering beams, the extra path difference introduced is

$$\Delta p = \text{Length } t \text{ in transparent medium} - \text{Length } t \text{ in air}$$

$$= \mu t - t = (\mu - 1)\,t$$

Suppose the present position of the particular fringe is

$$x = \frac{Dp}{d}$$

Then the new position of the same fringe will be

$$x' = \frac{D}{d}(p + \Delta p)$$

Hence the lateral displacement of the particular fringe on the screen is

$$\Delta x = x' - x = \frac{D\,\Delta p}{d}$$

or

$$\Delta x = \frac{D}{d}(\mu - 1)\,t = \frac{\beta}{\lambda}(\mu - 1)\,t$$

$$\left[\beta = \frac{D\lambda}{d} \quad \therefore \frac{D}{d} = \frac{\beta}{\lambda}\right]$$

As the shift is independent of n, every fringe (including the central fringe) or the entire fringe system is laterally displaced by Δx.

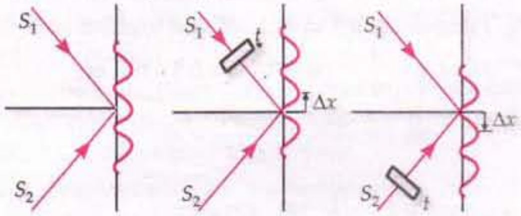


Fig. 10.17 Displacement of interference fringes due to the insertion of a thin glass sheet.

As shown in Fig. 10.17, the entire fringe system is shifted towards that side in which the thin transparent sheet is introduced. But there is no change in the fringe width.

Examples based on Displacement of Interference Fringes

Formulae Used

1. When a thin transparent sheet of thickness t and refractive index μ is inserted in one of the interfering beams, path difference introduced,
$$p = (\mu - 1)t$$
2. Displacement of the central bright fringe,
$$\Delta x = \frac{\beta}{\lambda}(\mu - 1)t = \frac{D}{d}(\mu - 1)t.$$

Units Used

Fringe width β, wavelength λ, thickness t and distances D, d are in metre.

Example 42. *Fringes are produced with monochromatic light of wavelength 5.45×10^{-5} cm. A thin glass plate of refractive index 1.5 is then placed normally in the path of one of the interference beams and the central bright band of the fringe system is found to move into the position previously occupied by the third bright band from the system. Find the thickness of the glass plate.*

Solution. Extra path difference introduced due to the insertion of glass plate,

$$p = \text{Length } t \text{ in glass plate} - \text{Length } t \text{ in air}$$
$$= \mu t - t = t(\mu - 1)$$

As the central bright fringe shifts into the position of third bright fringe, therefore,

$$(\mu - 1)t = 3\lambda$$

$\therefore$ Thickness,

$$t = \frac{3\lambda}{\mu - 1} = \frac{3 \times 5.45 \times 10^{-5}}{1.5 - 1} \text{ cm}$$
$$= \mathbf{32.7 \times 10^{-5}\ cm.}$$

Example 43. *A two slit Young's interference experiment is done with monochromatic light of wavelength 6000 Å. The slits are 2 mm apart and fringes are observed on a screen placed 10 cm away from the slits and it is found that the interference pattern shifts by 5 mm, when a transparent plate of thickness 0.5 mm is introduced in the path of one of the slits. What is the refractive index of the transparent plate?* [Roorkee 85]

Solution. Extra path difference introduced due to insertion of glass plate of thickness t,

$$p = (\mu - 1)t$$

$\therefore$ By using, $(\mu - 1)t = n\lambda = n.\frac{\beta d}{D}$ $\left(\because \beta = \frac{D\lambda}{d}\right)$

we get the fringe shift, $\Delta x = n\beta = \frac{D}{d}(\mu - 1)t$

Now $d = 2\text{ mm} = 2 \times 10^{-3}\text{m}$, $D = 10\text{ cm} = 0.1\text{m}$, $\lambda = 6000\text{ Å} = 6 \times 10^{-7}\text{ m}$, $\Delta x = 5\text{ mm} = 5 \times 10^{-3}\text{m}$, $t = 0.5\text{ mm} = 0.5 \times 10^{-3}\text{m}$

$$\therefore \quad \mu - 1 = \frac{\Delta x . d}{D . t} = \frac{5 \times 10^{-3} \times 2 \times 10^{-3}}{0.1 \times 0.5 \times 10^{-3}} = 0.2$$

Hence $\mu = \mathbf{1.2}$.

Example 44. *Monochromatic light of wavelength 600 nm is used in a Young's double slit experiment. One of the slits is covered by a transparent sheet of thickness 1.8×10^{-5} m made of a material of refractive index 1.6. How many fringes will shift due to the introduction of the sheet ?*

Solution. Extra path difference introduced due to insertion of glass plate of thickness t,

$$p = (\mu - 1)t$$

If the insertion of glass plate causes shift of n bright fringes, then

$$p = (\mu - 1)t = n\lambda$$

$$\therefore \quad n = \frac{(\mu - 1)t}{\lambda} = \frac{(1.6 - 1) \times 1.8 \times 10^{-5}}{600 \times 10^{-9}} = \mathbf{18.}$$

Problems For Practice

1. Find the thickness of a plate which will produce a change in optical path equal to half the wavelength λ of the light passing through it normally. The refractive index of the plate is μ. $\left(\textbf{Ans. } \frac{\lambda}{2(\mu - 1)}\right)$

2. When a thin sheet of a transparent material of thickness 7.2×10^{-4} cm is introduced in the path of one of the interfering beams, the central fringe shifts to a position occupied by the sixth bright fringe. If $\lambda = 6 \times 10^{-5}$cm, find the refractive index of the sheet. (Ans. 1.5)

3. In Young's double slit experiment, on inserting a thin plate of glass in the path of one of the interfering beams, it is found that the central bright fringe shifts into the position perviously occupied by the 6th bright fringe. If the wavelength of light used is 6×10^{-5} cm and the refractive index of glass plate is 1.5 for this wavelength, calculate the thickness of the plate. (Ans. 7.2×10^{-6} m)
4. A transparent paper ($\mu = 1.45$) of thickness 0.02 mm is pasted on one of the slits of a Young's double slit experiment which uses monochromatic light of wavelength 620 nm. How many fringes will cross the centre if the paper is removed ? (Ans. 14.5)
5. A glass plate of 1.2×10^{-6} m thickness is placed in the path of one of the interfering beams in a biprism arrangement using monochromatic light of wavelength 6000 Å. If the central band shifts by a distance equal to the width of the bands, find the refractive index of glass. (Ans. 1.5)

HINTS

1. $p = (\mu - 1)t = \dfrac{\lambda}{2} \quad \therefore \quad t = \dfrac{\lambda}{2(\mu - 1)}$
2. Here $\lambda = 6 \times 10^{-5}$cm, $t = 7.2 \times 10^{-4}$cm

   Displacement of the central bright fringe,

   $$\Delta x = 6\beta$$

   But $\Delta x = \dfrac{\beta}{\lambda}(\mu - 1)t$

   $\therefore \quad \dfrac{\beta}{\lambda}(\mu - 1)t = 6\beta$

   or $\mu - 1 = \dfrac{6\lambda}{t} = \dfrac{6 \times 6 \times 10^{-5}}{7.2 \times 10^{-4}} = 0.5$

   Refractive index, $\mu = 0.5 + 1 = \mathbf{1.5}$
3. $t = \dfrac{n\lambda}{\mu - 1} = \dfrac{6 \times 6 \times 10^{-7}}{1.5 - 1} = \mathbf{7.2 \times 10^{-6}\ m}$.
4. $n = \dfrac{(\mu - 1)t}{\lambda} = \dfrac{(1.45 - 1) \times 0.02 \times 10^{-3}}{620 \times 10^{-9}} \approx \mathbf{14.5}$.
5. Here $\Delta x = \beta$ or $\dfrac{D}{d}(\mu - 1)t = \dfrac{D\lambda}{d}$

   or $\mu - 1 = \dfrac{\lambda}{t} = \dfrac{6000 \times 10^{-10}}{1.2 \times 10^{-6}} = 0.5$

   $\therefore \quad \mu = 1 + 0.5 = \mathbf{1.5}$.

10.20 DIFFRACTION OF LIGHT

21. *What is diffraction of light ? Give some simple experiments illustrating diffraction of light.*

Diffraction of light. Light travels in a straight line. However, when light passes through a small hole, there is a certain amount of spreading of light. Similarly, when light passes by an obstacle, it appears to bend round the edges of the obstacle and enters its geometrical shadow.

The phenomenon of bending of light around the corners of small obstacles or apertures and its consequent spreading into the regions of geometrical shadow is called ***diffraction of light***.

To understand diffraction more clearly, consider a narrow aperture AB illuminated with light from a source S, as shown in Fig. 10.18(a). XY is a screen placed at large distance from AB. According to rectilinear propagation of light, only the portion $A'B'$ of the screen should be illuminated. However, it is seen that light enters the region of the geometrical shadow beyond A' and B'. The shadow is not sharp.

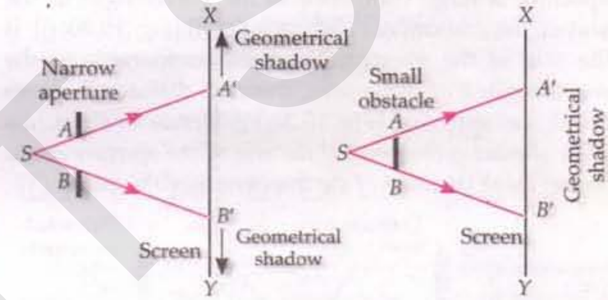


Fig. 10.18 Diffraction of light round corners of (a) a small aperture (b) a small object.

Similarly, when an obstacle AB ($e.g.$, a very small disc) is placed in the path of light, we expect a dark shadow $A'B'$ on the screen, as shown in Fig. 10.18(b). However, we observe a circular bright band at the centre, surrounded by dark and bright rings alternately. This shows that light bends around the edges, *i.e.*, light shows diffraction.

Experiments : **1.** As shown in Fig. 10.19, hold two blades so that their edges are parallel and form a narrow slit in between. Look through the slit on the straight filament of a clear glass bulb. With slight adjustment of the slit, a diffraction pattern of alternate bright and dark bands is seen.

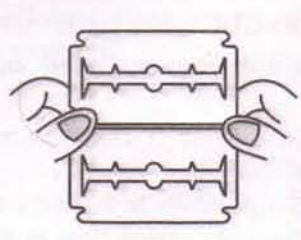


Fig. 10.19 A single slit formed by two blades.

2. Look at a street lamp through a piece of fine cloth. The lamp appears as an enlarged disc. The threads in mutually perpendicular directions enclose a number of slits which form a pattern of several weaker images of the slits.

3. A pinhole placed at a distance of 2m from a sodium lamp forms alternate bright and dark bands on a screen placed behind the pinhole.

22. *What should be the approximate size of the aperture or the obstacle to observe diffraction of light with it ? Explain it with the help of a simple activity.*

Size of aperture or obstacle for observing diffraction. Suppose plane waves are made to fall on a screen having a small aperture. The waves emerging out of the aperture are observed to be slightly curved at the edges. This is diffraction. If the size of the aperture is large compared to the wavelength of the waves, the amount of bending is small [Fig. 10.20(*a*)]. If the size of the aperture is small, comparable to the wavelength λ of the waves, then the diffracted waves are almost spherical [Fig. 10.20(*b*)]. Hence *the diffraction effect is more pronounced if the size of the aperture or the obstacle is of the order of the wavelength of the waves.*

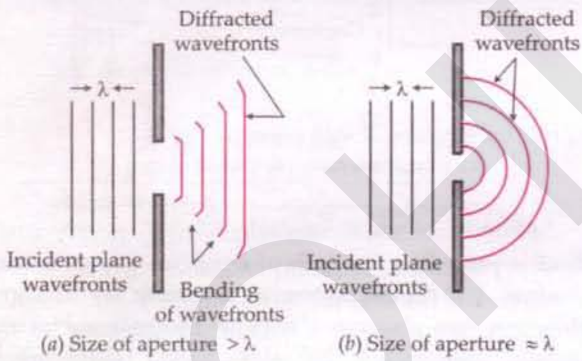


Fig. 10.20 Diffraction of a wave at a small aperture.

As the wavelength of light ($\approx 10^{-6}$ m) is much smaller than the size of the objects around us, so diffraction of light is not easily seen. But sound waves have large wavelength. They get easily diffracted by the objects around us.

10.21 FRESNEL AND FRAUNHOFFER DIFFRACTION

23. *Distinguish between Fresnel and Fraunhoffer diffraction.*

Two types of diffraction. The diffraction phenomena can be divided into *two* categories :

1. ***Fresnel's diffraction.*** In Fresnel's diffraction, the source and screen are placed close to the aperture or the obstacle and light after diffraction appears converging towards the screen and hence no lens is required to observe it. The incident wave fronts are either spherical or cylindrical.

2. ***Fraunhoffer's diffraction.*** In Fraunhoffer's diffraction, the source and screen are placed at large distances (effectively at infinity) from the aperture or the obstacle and converging lens is used to observe the diffraction pattern. The incident wavefront is planar one.

10.22 DIFFRACTION AT A SINGLE SLIT

24. *Explain the phenomenon of diffraction of light at a single slit to show the formation of diffraction fringes. Show graphically the variation of intensity with angle in this diffraction pattern. Why secondary maxima are less intense than the central maximum ?*

Diffraction at a single slit. As shown in Fig. 10.21, a source S of monochromatic light is placed at the focus of a convex lens L_1. A parallel beam of light and hence a plane wavefront WW gets incident on a narrow rectangular slit AB of width a.

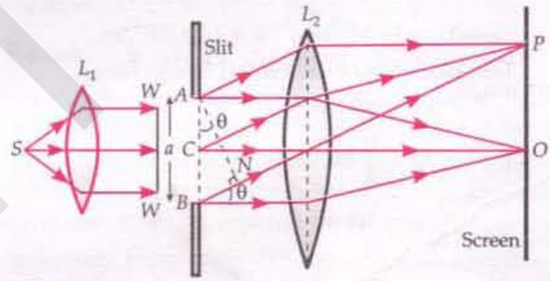


Fig. 10.21 Diffraction at a single slit.

The incident wavefront disturbs all parts of the slit AB simultaneously. According to Huygens' theory, all parts of the slit AB will become source of secondary wavelets, which all start in the same phase. These wavelets spread out in all directions, thus causing diffraction of light after it emerges through slit AB. Suppose the diffraction pattern is focussed by a convex lens L_2 on a screen placed in its focal plane.

Central maximum. All the secondary wavelets going straight across the slit AB are focussed at the central point O of the screen. The wavelets from any two corresponding points of the two halves of the slit reach the point O in the same phase, they add constructively to produce a *central bright fringe*. For detailed explanation of diffraction fringes, see *For Your Knowledge* box on page 10.32.

Calculation of path difference. Suppose the secondary wavelets diffracted at an angle θ are focussed at point P. The secondary wavelets start from different parts of the slit in same phase but they reach the point P in different phases. Draw perpendicular AN from A on to the ray from B. Then the path difference between the wavelets from A and B will be

$$p = BP - AP = BN = AB \sin\theta = a \sin\theta.$$

Positions of minima. Let the point P be so located on the screen that the path difference, $p = \lambda$ and angle $\theta = \theta_1$. Then from the above equation, we get

$$a \sin\theta_1 = \lambda$$

We can divide the slit AB into two halves AC and CB. Then the path difference between the wavelets from A and C will be $\frac{\lambda}{2}$. Similarly, corresponding to every point in the upper half AC, there is a point in the lower half CB for which the path difference is $\frac{\lambda}{2}$. Hence the wavelets from the two halves reach the point P always in opposite phases. They interfere destructively so as to produce a minimum.

Thus the condition for *first dark fringe* is

$$a \sin\theta_1 = \lambda$$

Similarly, the condition for *second dark fringe* will be

$$a \sin\theta_2 = 2\lambda$$

Hence the condition for ***nth dark fringe*** can be written as

$$a \sin\theta_n = n\lambda, \quad n = 1, 2, 3, ...$$

The directions of various minima are given by

$$\theta_n \approx \sin\theta_n = n\frac{\lambda}{a}$$

[As $\lambda << a$, so $\sin\theta_n \approx \theta_n$]

Positions of secondary maxima. Suppose the point P is so located that $p = \frac{3\lambda}{2}$

When $\theta = \theta'_1$, then $a \sin\theta'_1 = \frac{3}{2}\lambda$

We can divide the slit into three equal parts. The path difference between two corresponding points of the first two parts will be $\frac{\lambda}{2}$. The wavelets from these points will interfere destructively. However, the wavelets from the third part of the slit will contribute to some intensity forming a secondary maximum. The intensity of this maximum is much less than that of the central maximum.

Thus the condition for the *first secondary maximum* is

$$a \sin\theta'_1 = \frac{3}{2}\lambda$$

Similarly, the condition for the ***second secondary maximum*** is

$$a \sin\theta'_2 = \frac{5}{2}\lambda$$

Hence the condition for ***nth secondary maximum*** can be written as

$$a \sin\theta'_n = (2n+1)\frac{\lambda}{2}, n = 1, 2, 3, ...$$

The directions of secondary maxima are given by

$$\theta'_n \approx \sin\theta'_n = (2n+1)\frac{\lambda}{2a}$$

The intensity of secondary maxima decreases as n increases.

Intensity distribution curve. If we plot a graph between the intensities of maxima and minima against the diffraction angle θ, we get a graph of the type shown in Fig. 10.22. It has a broad central maximum in the direction ($\theta = 0°$) of incident light. On either side of the central maximum, it has secondary maxima of decreasing intensity at positions,

$$\theta = \pm(2n+1)\frac{\lambda}{2a}$$

and minima at positions,

$$\theta = \pm n\frac{\lambda}{a} \qquad (n \neq 0).$$

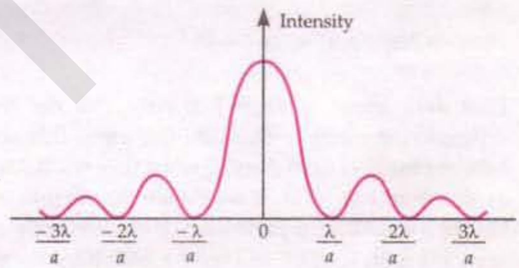


Fig. 10.22 Variation of intensity with angle θ in single slit diffraction.

The intensities of secondary maxima relating to the intensity of central maximum are in ratio,

$$1 : \frac{1}{21} : \frac{1}{61} : \frac{1}{121} .....$$

Thus the intensity of the first secondary maximum is just 4% of that of the central maximum.

Intensity of secondary maxima decreases with the order of the maximum. The reason is that the intensity of the central maximum is due to the constructive interference of wavelets from all parts of the slit, the first secondary maximum is due to the contribution of wavelets from one third part of the slit (wavelets from remaining two parts interfere destructively), the second secondary maximum is due to the contribution of wavelets from the one fifth part only (the remaining four parts interfere destructively) and so on. Hence the intensity of secondary maximum decreases with the increase in the order n of the maximum.

For Your Knowledge

➢ **Explanation of diffraction fringes**

Central maximum. All the wavelengths going straight ($\theta = 0°$) across the slit are focussed at the central point O of the screen, as shown in Fig. 10.23. The wavelets from any two corresponding points such as (0, 12), (2, 10), (4, 8) etc. from the two halves of the slit have zero path difference. They undergo constructive interference to produce central bright fringe.

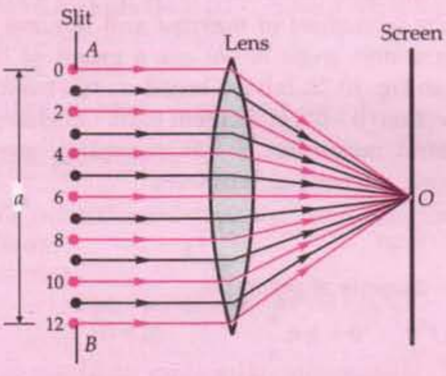


Fig. 10.23 Diffraction at angle $\theta = 0°$.

First dark fringe. If angle θ is such that the path difference, $p = a \sin \theta = \lambda$, then the path difference between the rays from A and B when they reach P is λ, as shown in Fig. 10.24. If we divide the slit into two halves I and II, of 6 parts each, then obviously the wavelets form 0 and 6 will have a path difference of $\lambda/2$ or a phase difference of π. They interfere destructively. Similarly, the wavelet pairs (1, 7), (2, 8), (3, 9), (4, 10), (5, 11) and (6, 12) of the two halves will interfere destructively. Hence the condition for first dark fringe is

$$a \sin \theta = \lambda$$

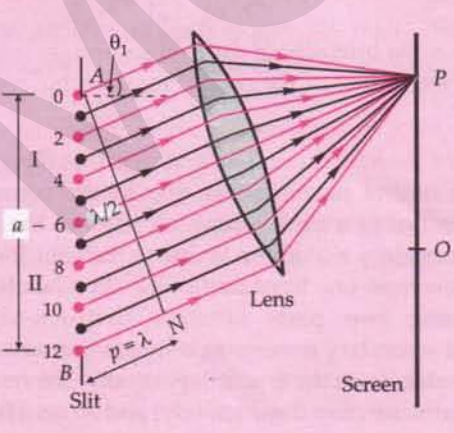


Fig. 10.24 Diffraction at an angle θ given by $a \sin\theta = \lambda$.

First secondary maximum. Suppose the angle θ is such that the path difference, $p = a \sin \theta = 3\lambda/2$.

We can divide the slit into three equal regions I, II and III, as shown in Fig. 10.25. The path difference between any two corresponding points of regions I and II will be $\frac{\lambda}{2}$ or phase difference will be π. The wavelets from these points will interfere destructively. The wavelets from III region of the slit will contribute to some intensity forming a secondary maximum. The intensity of this maximum is much less than the central maximum. The condition for the first secondary maximum can be written as

$$a \sin \theta = \frac{3}{2} \lambda$$

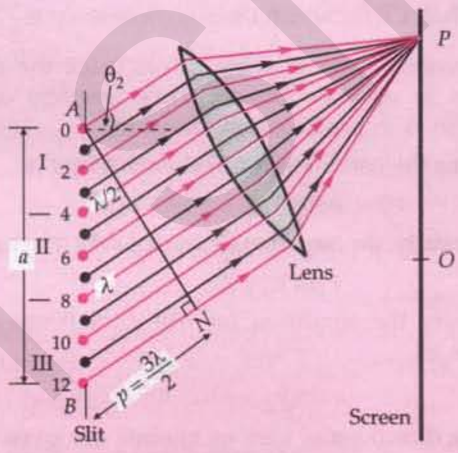


Fig. 10.25 Diffraction at an angle θ given by $a \sin\theta = 3/2\ \lambda$.

10.23 WIDTHS OF CENTRAL AND SECONDARY MAXIMA

25. *Deduce expressions for (i) angular width of central maximum (ii) linear width of central maximum and (iii) linear width of a secondary maximum.*

Angular width of central maximum. *The angular width of the central maximum is the angular separation between the directions of the first minima on the two sides of the central maximum*, as shown in Fig. 10.26.

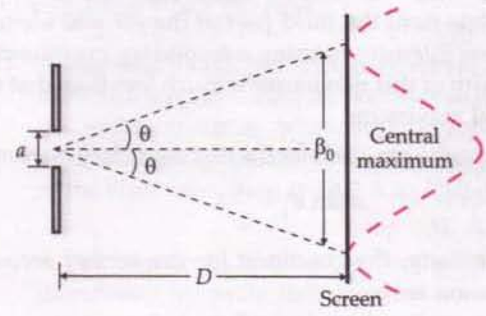


Fig. 10.26 Angular and linear widths of central maximum.

The directions of first minima on either side of central maximum are given by

$$\theta = \frac{\lambda}{a}$$

This angle is called ***half angular width of central maximum***.

$\therefore$ Angular width of central maximum $= 2\theta = \dfrac{2\lambda}{a}$

Linear width of central maximum. If D is the distance of the screen from the single slit, then the linear width of central maximum will be

$$\beta_0 = D \times 2\theta = \frac{2D\lambda}{a}$$

$$\left[2\theta\,(\text{rad}) = \frac{\text{Arc}}{\text{Radius}} = \frac{\beta_0}{D}\right]$$

Linear width of a secondary maximum. *The angular width of nth secondary maximum is the angular separation between the directions of nth and (n + 1)th minima.*

Direction of nth minimum, $\theta_n = n\dfrac{\lambda}{a}$

Direction of $(n+1)$ th minimum,

$$\theta_{n+1} = (n+1)\frac{\lambda}{a}$$

$\therefore$ Angular width of nth secondary maximum

$$= \theta_{n+1} - \theta_n = (n+1)\frac{\lambda}{a} - n\frac{\lambda}{a} = \frac{\lambda}{a}$$

Hence the linear width of nth secondary maximum

$$= \text{Angular width} \times D$$

or $$\beta = \frac{D\lambda}{a}$$

Clearly, $\beta_0 = 2\beta$

Thus the central maximum of a diffraction pattern is twice as wide as any secondary maximum.

Clearly, width of a secondary maximum

$$\propto \frac{1}{\text{slit width}}$$

As the slit width is increased, the secondary maxima get narrower. If the slit is sufficiently wide, the secondary maxima disappear and only the central maximum is obtained which is the sharp image of the slit. Thus a distinct diffraction pattern is possible only if the slit is very narrow.

10.24 VALIDITY OF RAY OPTICS : FRESNEL'S DISTANCE

26. *Geometrical optics is based upon the rectilinear propagation of light. Diffraction effects show that light does not travel in straight lines. Under what conditions the concepts of ray optics are valid ? Define Fresnel's distance and size of Fresnel's zone.*

Ray optics as a limiting case of wave optics : Fresnel's distance and Fresnel's zone. A parallel beam of light of wavelength λ on passing through an aperture of size a gets diffracted into a beam of angular width,

$$\theta = \frac{\lambda}{a}$$

If a screen is placed at distance D, this beam spreads over a linear width,

$$x = \frac{D\lambda}{a}$$

If the diffraction spread x is small, the concept of ray optics will be valid.

If we have an aperture of size $a = 10$ mm and use light of wavelength $\lambda = 6 \times 10^{-7}$ m, then the beam after travelling a distance of 3 m will get diffracted through a width

$$x = \frac{D\lambda}{a} = \frac{3 \times 6 \times 10^{-7}}{10 \times 10^{-3}}$$

$$= 18 \times 10^{-5}\ \text{m} = 0.18\ \text{mm}$$

This diffraction spreading is not quite large. Thus ray optics is valid in many common situations. It is useful here to define what is called Fresnel's distance, D_F.

The distance at which the diffraction spread of a beam is equal to the size of the aperture is called ***Fresnel's distance***.

i.e., when $x = a$, $D = D_F$

$$\therefore \quad a = \frac{D_F \lambda}{a} \quad \text{or} \quad D_F = \frac{a^2}{\lambda}$$

If $D < D_F$, then there will not be too much broadening by diffraction *i.e.*, the light will travel along straight lines and the concepts of ray optics will be valid.

$$\text{As } D < D_F \quad \text{or} \quad D < \frac{a^2}{\lambda} \quad \text{or} \quad a > \sqrt{\lambda D}$$

For a given value of D, the quantity $\sqrt{\lambda D}$ is called the ***size of Fresnel zone*** and is denoted by a_F.

i.e. $$a_F = \sqrt{\lambda D}$$

Hence the concepts of ray optics can be conveniently used without introducing any appreciable error if the size of the aperture is greater than the size of the Fresnel zone,

i.e., $$a > a_F.$$

Examples based on Diffraction of Light and Fresnel's distance

Formulae Used

1. For diffraction at a single slit of width a,
   (i) Condition for nth minimum is
   $$a \sin\theta = n\lambda, \quad \text{where } n = 1, 2, 3, ...$$
   (ii) Condition of nth secondary maximum is
   $$a \sin\theta = (2n+1)\frac{\lambda}{2}, \quad \text{where } n = 1, 2, 3, ...$$
   (iii) Angular position or direction of nth minimum,
   $$\theta_n = \frac{n\lambda}{a}$$
   (iv) Distance of nth minimum from the centre of the screen,
   $$x_n = \frac{nD\lambda}{a}$$
   (v) Angular position of nth secondary maximum,
   $$\theta'_n = (2n+1)\frac{\lambda}{2a}$$
   (vi) Distance of nth secondary maximum from the centre of the screen,
   $$x'_n = (2n+1)\frac{D\lambda}{2a}$$
   (vii) Width of central maximum, $\beta_0 = 2\beta = \frac{2D\lambda}{a}$
   (viii) Angular spread of central maximum on either side, $\theta = \pm\frac{\lambda}{a}$
   (ix) Total angular spread of central maximum,
   $$2\theta = \frac{2\lambda}{a}$$
2. For diffraction at a circular aperture of diameter d,
   (i) Angular spread of central maximum,
   $$\theta = \frac{1.22\lambda}{a}$$
   (ii) Linear spread, $x = D\theta$
   (iii) Areal spread, $x^2 = (D\theta)^2$
   where D is the distance at which the effect is considered.
3. Fresnel distance, $D_F = \frac{a^2}{\lambda}$
4. Size of Fresnel zone, $a_F = \sqrt{\lambda D}$.

Units Used

Angles θ, θ_n and θ'_n are in radian, wavelength λ in metre, distances a, a_F, D and D_F in metre and n is a pure number.

Example 45. *Fraunhoffer diffraction from a single slit of width 1.0 μm is observed with light of wavelength 500 nm. Calculate the half angular width of the central maximum.* [ISCE 2000]

Solution. The Fraunhoffer diffraction is the diffraction of plane wavefronts from a single slit.

Here $a = 1.0\ \mu m = 1.0 \times 10^{-6}$ m,

$\lambda = 500$ nm $= 500 \times 10^{-9}$ m

Half angular width θ of the central maximum is given by

$$\sin\theta = \frac{\lambda}{a} = \frac{500 \times 10^{-9}}{1.0 \times 10^{-6}} = 0.5$$

$$\therefore \quad \theta = \mathbf{30°}.$$

Example 46. *Light of wavelength 600 nm falls normally on a slit of width 1.2 μm producing Fraunhoffer diffraction pattern on a screen. Calculate the angular position of the first minimum and the angular width of the central maximum.* [ISCE 02]

Solution. Here $\lambda = 600$ nm $= 600 \times 10^{-9}$ m,

$a = 1.2\ \mu m = 1.2 \times 10^{-6}$ m

The angular position θ of the first dark fringe is given by

$$\sin\theta = \frac{\lambda}{a} = \frac{600 \times 10^{-9}}{1.2 \times 10^{-6}} = \frac{1}{2}$$

$$\therefore \quad \theta = \mathbf{30°}.$$

Angular width of central maximum $= 2\theta = 60°$.

Example 47. *Microwaves of frequency 24,000 MHz are incident normally on a rectangular slit of width 5 cm. Calculate the angular spread of the central maximum of the diffraction pattern of the slit.*

Solution. Here $f = 24{,}000$ MHz $= 24 \times 10^9$ Hz,

$a = 5$ cm $= 5 \times 10^{-2}$ m

Angular spread of central maximum is

$$2\theta = \frac{2\lambda}{a} = \frac{2c}{a\nu} = \frac{2 \times 3 \times 10^8}{5 \times 10^{-2} \times 24 \times 10^9} = \mathbf{\frac{1}{2}\ rad.}$$

Example 48. *A slit of width 'a' is illuminated by red light of wavelength 6500 Å. For what value of 'a' will (i) the first minimum fall at an angle of diffraction of 30° and (ii) the first maximum fall at an angle of diffraction of 30°.* [CBSE OD 09]

Solution. (i) For first minimum of the diffraction pattern,

$$a \sin\theta = \lambda$$

$$\therefore \quad a = \frac{\lambda}{\sin\theta} = \frac{6{,}500 \times 10^{-10}\ \text{m}}{\sin 30°}$$

$$= \frac{6{,}500 \times 10^{-10}\ \text{m}}{0.5} = \mathbf{1.3 \times 10^{-6}\ m.}$$

(*ii*) For first secondary maximum of the diffraction pattern,

$$a \sin\theta = \frac{3\lambda}{2}$$

$$\therefore \quad a = \frac{3\lambda}{2\sin\theta} = \frac{3 \times 6{,}500 \times 10^{-10}\,\text{m}}{2 \times \sin 30^\circ}$$

$$= \mathbf{1.95 \times 10^{-6}\,m}.$$

Example 49. *Light of wavelength 550 nm is incident as parallel beam on a slit of width 0.1 mm. Find the angular width and the linear width of the principal maxima in the resulting diffraction pattern on a screen kept at a distance of 1.1 m from the slit. Which of these widths would not change if the screen were moved to a distance of 2.2 m from the slit ?* [CBSE Sample Paper 08]

Solution. Here $\lambda = 550\text{ nm} = 550 \times 10^{-9}\,\text{m}$,
$a = 0.1\text{ mm} = 0.1 \times 10^{-3}\,\text{m}$, $D = 1.1\text{ m}$

Angular width of principal maximum,

$$2\theta = \frac{2\lambda}{a} = \frac{2 \times 550 \times 10^{-9}}{0.1 \times 10^{-3}} = \mathbf{1.1 \times 10^{-2}\,rad}$$

Linear width of principal maximum,

$$\beta_0 = \frac{2D\lambda}{a} = 1.1 \times 0.011 = 0.0121\text{ m} = \mathbf{12.1\text{ mm}}.$$

When the screen is moved to a distance of 2.2 m, the angular width would not change because it is independent of this distance D.

Example 50. *A parallel beam of light of 600 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1.2 m away. It is observed that the first minimum is at a distance of 3 mm from the centre of the screen. Calculate the width of the slit.* [CBSE OD 13]

Solution. Here $\lambda = 600\text{nm} = 6 \times 10^{-7}\,\text{m}$, $D = 1.2\text{ m}$,
$x_1 = 3\text{mm} = 3 \times 10^{-3}\,\text{m}$

Position of first minimum, $x_1 = \frac{D\lambda}{a}$

Slit width, $a = \frac{D\lambda}{x_1} = \frac{1.2 \times 6 \times 10^{-7}}{3 \times 10^{-3}}$

$$= 2.4 \times 10^{-4}\text{ m} = \mathbf{0.24\text{ mm}}.$$

Example 51. *A parallel beam of light of wavelength 600 nm is incident normally on a slit of width 'a'. If the distance between the slits and the screen is 0.8 m and the distance of 2nd order maximum from the centre of the screen is 15 mm, calculate the width of the slit.* [CBSE OD 08]

Solution. Distance of 2^{nd} order maximum from the centre of the screen,

$$x_2' = \frac{5}{2}\frac{D\lambda}{a} \quad \text{or} \quad a = \frac{5}{2}\frac{D\lambda}{x_2'}$$

Given $\lambda = 600\text{ nm} = 6 \times 10^{-7}\,\text{m}$, $D = 0.8\text{ m}$,
$x_2' = 15\text{ mm} = 15 \times 10^{-3}\text{ m}$

$$\therefore \quad a = \frac{5 \times 0.8 \times 6 \times 10^{-7}}{2 \times 15 \times 10^{-3}}$$

$$= 8 \times 10^{-5}\,\text{m} = \mathbf{80\,\mu m}.$$

Example 52. *Determine the angular separation between central maximum and first order maximum of the diffraction pattern due to a single slit of width 0.25 mm when light of wavelength 5890 Å is incident on it normally.* [CBSE OD 93]

Solution. From Fig. 10.27, it is clear that the angular separation between central maximum and first order minimum is

$$\theta = \frac{3\lambda}{2a} - 0 = \frac{3\lambda}{2a}$$

or

$$\theta = \frac{3 \times 5890 \times 10^{-10}}{2 \times 0.25 \times 10^{-3}} = \mathbf{3.534 \times 10^{-3}\ rad}.$$

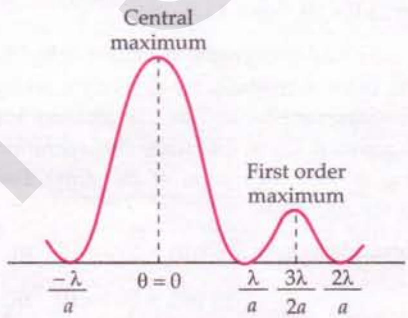


Fig. 10.27

Example 53. *Parallel light of wavelength 5000 Å falls normally on a single slit. The central maximum spreads out to 30° on either side of the incident light. Find the width of the slit. For what width of the slit the central maximum would spread out to 90° from the direction of the incident light ?*

Solution. Angular spread of central maximum on either side of incident light is given by

$$\sin\theta = \frac{\lambda}{a}$$

∴ Slit width,

$$a = \frac{\lambda}{\sin\theta} = \frac{5000 \times 10^{-10}}{\sin 30^\circ} = 10^{-6}\,\text{m}$$

For $\theta = 90^\circ$, we have

$$a = \frac{\lambda}{\sin 90^\circ} = \frac{5000 \times 10^{-10}}{1} = \mathbf{5 \times 10^{-7}\,m}.$$

Example 54. *A slit of width 0.025 mm is placed in front of a lens of focal length 50 cm. The slit is illuminated with light* *of wavelength 5900 Å. Calculate the distance between the centre and first dark band of diffraction pattern obtained on a screen placed at the focal plane of the lens.*

Solution. Here $\lambda = 5900\,\text{Å} = 59\times10^{-8}\text{m}$,

$f = 50\text{ cm} = 0.50\text{ m},\ a = 0.025\text{ mm} = 2.5\times10^{-5}\text{m}$

For first dark band, $\sin\theta = \frac{\lambda}{a}$

As the diffraction pattern is obtained in the focal plane of lens, therefore

$$\tan\theta = \frac{x}{f}$$

where x is the distance between the centre and the first dark band.

For small θ, $\tan\theta \simeq \sin\theta$ or $\frac{x}{f} = \frac{\lambda}{a}$

$$\therefore\quad x = \frac{\lambda}{a}\times f = \frac{59\times10^{-8}\times0.50}{2.5\times10^{-5}}$$

$$= 11.8\times10^{-3}\text{m} = \mathbf{11.8\ mm.}$$

Example 55. *Two wavelengths of sodium light 590 nm, 596 nm are used, in turn, to study the diffraction taking place at a single slit of aperture 2×10^{-6} m. The distance between the slit and the screen is 1.5 m. Calculate the separation between the positions of first maximum of the diffraction pattern obtained in the two cases.* [CBSE D 06C, 13]

Solution. Here $\lambda_1 = 590\,\text{nm} = 590\times10^{-9}\text{m}$

$$\lambda_2 = 596\text{ nm} = 596\times10^{-9}\text{m},$$
$$a = 2\times10^{-6}\text{m},\quad D = 1.5\text{ m}$$

Distance of first secondary maximum from the centre of the screen is

$$x = \frac{3}{2}\frac{D\lambda}{a}$$

For the two wavelengths, we have

$$x_1 = \frac{3}{2}\frac{D\lambda_1}{a}\quad\text{and}\quad x_2 = \frac{3}{2}\frac{D\lambda_2}{a}$$

Spacing between the first two maximum of sodium lines

$$= x_2 - x_1 = \frac{3D}{2a}(\lambda_2 - \lambda_1)$$

$$= \frac{3\times1.5}{2\times2\times10^{-6}}(596\times10^{-9} - 590\times10^{-9})$$

$$= \frac{3\times1.5\times6\times10^{-3}}{4}$$

$$= 6.75\times10^{-3}\text{m} = \mathbf{6.75\ mm}$$

Example 56. *Estimate the angular separation between first order maximum and third order minimum of the diffraction pattern due to a single slit of width 1 mm, when light of wavelength 600 nm is incident normal on it.* [CBSE OD 15C]

Solution. Angular position of nth maximum,

$$\theta'_n = (2n+1)\frac{\lambda}{2a}$$

Angular position of first order maximum, $\theta'_1 = \frac{3\lambda}{2a}$

Angular position of nth minimum, $\theta_n = \frac{n\lambda}{a}$

Angular position of 3rd order minimum, $\theta_3 = \frac{3\lambda}{a}$

∴ Required angular separation

$$= \theta_3 - \theta'_1 = \frac{3\lambda}{2a} = \frac{3\times600\times10^{-9}}{2\times1\times10^{-3}} = \mathbf{9\times10^{-4}\ rad.}$$

Example 57. *A laser operates at a frequency of 3×10^{14} Hz and has an aperture of 10^{-2} m. What will be the angular spread?*

Solution. Here $\nu = 3\times10^{14}$ Hz, $a = 10^{-2}$m, $c = 3\times10^{8}\text{ms}^{-1}$

Wavelength,

$$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{3\times10^{14}} = 10^{-6}\text{m}$$

∴ Angular spread,

$$\theta = \frac{1.22\,\lambda}{a} = \frac{1.22\times10^{-6}}{10^{-2}} = \mathbf{1.22\times10^{-4}\ rad.}$$

Example 58. *A laser beam has a wavelength of 7×10^{-7} m and aperture 10^{-2} m. The beam is sent to moon, the distance of which from earth is 4×10^5 km. Find (i) the angular spread and (ii) areal spread when the beam reaches the moon.*

Solution. Here $\lambda = 7\times10^{-7}$m, $a = 10^{-2}$m, $D = 4\times10^5\text{ km} = 4\times10^8\text{ m}$

For the circular aperture, we have

(*i*) Angular spread,

$$\theta = \frac{1.22\,\lambda}{a} = \frac{1.22\times7\times10^{-7}}{10^{-2}} = \mathbf{8.54\times10^{-5}\ rad.}$$

(*ii*) Areal spread

$$= (D\theta)^2 = (4\times10^8\times8.54\times10^{-5})^2 = \mathbf{1.197\times10^9\ m^2.}$$

Example 59. *A laser light beam of power 20 mW is focused on a target by a lens of focal length 0.05 m. If the aperture of the laser be 1 mm and the wavelength of its light 7000 Å, calculate the angular spread of the laser, the area of the target hit by it, and the intensity of the impact on the target.*

Solution. Here $P = 20\text{ mW} = 20\times10^{-3}\text{W}$,

$$f = 0.05\text{ m},\quad a = 1\text{ mm} = 10^{-3}\text{m},$$
$$\lambda = 7000\text{ Å} = 7000\times10^{-10}\text{m}$$

(*a*) Angular spread of the laser beam,

$$\theta = \frac{1.22\,\lambda}{a} = \frac{1.22\times7000\times10^{-10}}{1\times10^{-3}}$$

$$= \mathbf{8.54\times10^{-4}\ radian.}$$

(*b*) Linear spread of the laser

$$= f.\theta = 5\times10^{-2}\times8.54\times10^{-4}\,\text{m}$$

$\therefore$ Areal spread of the laser, *i.e.*, area of the target hit by it

$$=(5\times8.54\times10^{-6})^2 = \mathbf{1.823\times10^{-9}\,m^2}.$$

(*c*) Intensity of impact of the laser on the target

$$=\frac{\text{Power of laser}}{\text{Area hit}}=\frac{20\times10^{-3}}{1.823\times10^{-9}}$$

$$= \mathbf{10.97\times10^{6}\ Wm^{-2}}.$$

Example 60. *For what distance is ray optics a good approximation when the aperture is 3 mm wide and the wavelength is 500 nm ?* [NCERT]

Solution. Here $a = 3\text{ mm} = 3\times10^{-3}\text{m}$,

$$\lambda = 500\text{ nm} = 500\times10^{-9}\text{m}$$

The distance upto which a beam of light can travel without significant broadening is called Fresnel distance and its value is given by

$$D_F=\frac{a^2}{\lambda}=\frac{(3\times10^{-3})^2}{500\times10^{-9}}=\mathbf{18\ m}.$$

Example 61. *Light of wavelength* 5×10^{-7} *m is diffracted by an aperture of width* 2×10^{-3} *m. For what distance travelled by the diffracted beam does the spreading due to diffraction become greater than the width of the aperture ?*

Solution. Fresnel distance,

$$D_F=\frac{a^2}{\lambda}=\frac{(2\times10^{-3})^2}{5\times10^{-7}}=\mathbf{8\ m}$$

So at a distance greater than 8 m, the spreading due to diffraction becomes greater than the width of the aperture.

Problems For Practice

1. A slit 4.0 cm wide is irradiated with microwaves of 2.0 cm. Find the angular spread of central maximum assuming incidence normal to the plane of the slit. (Ans. 60°)
2. The slit of width '*d*' is illuminated by light of wavelength 5000 Å. For what value of '*d*' will the first maximum fall at angle of diffraction of 30° ? [CBSE F 06] (Ans. 1.5×10^{-6}m)
3. The light of wavelength 600 nm is incident normally on a slit of width 3 mm. Calculate the linear width of central maximum on a screen kept 3 m away from the slit. [CBSE OD 01] (Ans. 1.2 mm)
4. Light, of wavelength 500 nm, falls, from a distant source, on a slit 0.50 mm wide. Find the distance between the two dark bands, on either side of the central bright band, of the diffraction pattern observed, on a screen placed 2 m from the slit. [CBSE.O 04C] (Ans. 4 mm)
5. A 0.02 cm wide slit is illuminated at normal incidence by light of wavelength 6000 Å (*i*) Find the width of the central maximum band on the screen placed 1 m away from the slit. (*ii*) What should be the fringe width if the apparatus is immersed in water whose refractive index is 4/3 ? [Ans. (*i*) 0.6 cm (*ii*) 0.45 cm]
6. A Fraunhoffer diffraction pattern due to a single slit of width 0.2 mm is being obtained on a screen placed at a distance of 2 m from the slit. The first minima lie at 5 mm on either side of the central maximum on the screen. Find the wavelength of light used. (Ans. 5000 Å)
7. A parallel beam of monochromatic light of wavelength 5000 Å is incident normally on a narrow slit of width 0.25 mm. The diffraction pattern is observed on a screen placed at the focal plane of a convex lens placed close to the slit between slit and screen. Find the angular separation between the first secondary maxima on either side of the central maximum. (Ans. 6×10^{-3} rad)
8. A screen is placed 50 cm from a single slit which is illuminated with light of wavelength 6000 Å. If the distance between the first and third minima in the diffraction pattern is 3.0 mm, what is the width of the slit ? (Ans. 0.2 mm)
9. The width of an aperture is 4 mm and wavelength is 5000 Å. Calculate the distance upto which ray optics is valid. (Ans. 32 m)
10. A parallel beam of light of wavelength 600 nm is incident normally on a slit of width '*d*'. If the distance between the slits and the screen is 0.8 m and the distance of 2nd order minimum from the centre of the screen is 9.5 mm, calculate the width of the slit. [CBSE OD 08] (Ans. 0.1 mm)
11. Two towers are built on hill 50 km apart and the line joining them passes 30 m above a hill halfway in between. What is the longest wavelength of radiowaves which can be sent between the towers without serious diffraction effects ? (Ans. 0.036 m)

HINTS

1. Here $a = 4.0$ cm, $\lambda = 2.0$ cm,

$$\sin\theta=\frac{\lambda}{a}=\frac{2.0}{4.0}=\frac{1}{2}\quad\text{or}\quad\theta=30^\circ$$

The central maximum spreads on both sides of the screen.

$\therefore$ Angular spread of central maximum = **60°**.

2. For first secondary maximum of the diffraction pattern,

$$a \sin \theta = \frac{3\lambda}{2}$$

$$\therefore \quad a = \frac{3\lambda}{2 \sin \theta} = \frac{3 \times 5000 \times 10^{-10}}{2 \times \sin 30^\circ} = \mathbf{1.5 \times 10^{-6}\ m.}$$

3. Linear width, $\beta_0 = 2\frac{D\lambda}{a} = \frac{2 \times 3 \times 600 \times 10^{-9}}{3 \times 10^{-3}}$

$$= 1.2 \times 10^{-3} \text{m} = \mathbf{1.2\ mm.}$$

4. $\beta_0 = \frac{2D\lambda}{a} = \frac{2 \times 2 \times 500 \times 10^{-9}}{0.50 \times 10^{-3}} = 4 \times 10^{-3} \text{m} = \mathbf{4\ mm.}$

5. (*i*) $\beta_0 = \frac{2D\lambda}{a} = \frac{2 \times 1 \times 6000 \times 10^{-10}}{0.02 \times 10^{-2}}$

$$= 6 \times 10^{-3} \text{m} = \mathbf{0.6\ cm.}$$

(*ii*) When the apparatus is immersed in water, wavelength becomes $\lambda' = \lambda / \mu$

$\therefore$ Fringe width, $\beta_0' = \frac{D\lambda'}{a} = \frac{\beta_0}{\mu} = \frac{0.6}{4/3} = \mathbf{0.45\ cm.}$

6. As $x_1 = \frac{D\lambda}{a}$

$$\therefore \quad \lambda = \frac{x_1 a}{D} = \frac{5 \times 10^{-3} \times 0.2 \times 10^{-3}}{2}$$

$$= 5 \times 10^{-7} \text{m} = \mathbf{5000\ Å.}$$

7. Angular position of first secondary maximum on either side of central maximum $\theta_1' = \frac{3\lambda}{2a}$

Total angular separation between the first secondary on either side is

$$2\theta_1' = \frac{3\lambda}{a} = \frac{3 \times 5000 \times 10^{-10}}{0.25 \times 10^{-3}} = \mathbf{6 \times 10^{-3}\ rad.}$$

8. Position of *n*th minimum in the diffraction pattern,

$$x_n = \frac{nD\lambda}{a}$$

$$\therefore \quad x_3 - x_1 = (3-1)\frac{D\lambda}{a} = \frac{2D\lambda}{a}$$

or $\quad a = \frac{2D\lambda}{x_3 - x_1} = \frac{2 \times 0.50 \times 6000 \times 10^{-10}}{3.0 \times 10^{-3}}$

$$= 2 \times 10^{-4} \text{m} = \mathbf{0.2\ mm.}$$

9. Find $D_F = \frac{a^2}{\lambda}$.

10. $x_2 = \frac{2\,D\lambda}{a}$

$$\therefore a = \frac{2D\lambda}{x_2} = \frac{2 \times 0.8 \times 600 \times 10^{-9}}{9.5 \times 10^{-3}} = \frac{96}{95} \times 10^{-4} \text{m}$$

$$\simeq \mathbf{0.1\ mm.}$$

11. Proceed as in Exercise 10.18 on page 10.85.

10.25 INTERFERENCE vs. DIFFRACTION

27. *Show that the fringe pattern obtained on the screen is actually a superposition of single-slit diffraction from each slit and the double-slit interference patterns.*

The actual double-slit interference pattern. In double-slit experiment, the pattern on the screen is actually a superposition of single-slit diffraction from each slit and the double-slit interference pattern. As shown in Fig. 10.28, the actual pattern shows a broader diffraction peak in which there appear several fringes

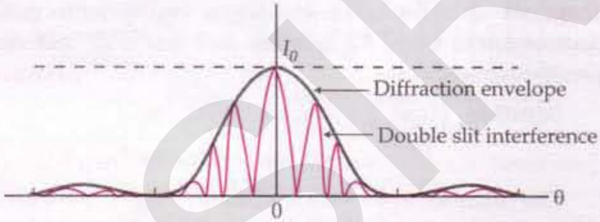


Fig. 10.28 The actual-double slit interference pattern.

of smaller width due to double-slit interference. The number of interference fringes depends on the ratio d/a = distance between the slits/slit width. When d/a becomes very small, the diffraction pattern becomes very flat and we observe the normal double-slit interference pattern as shown in Fig. 10.15.

28. *Give some important points of difference between interference and diffraction.*

	Interference	Diffraction
1.	Interference is the result of superposition of secondary waves starting from two different wavefronts originating from two coherent sources.	Diffraction is the result of superposition of secondary waves starting from different parts of the same wavefront.
2.	All bright and dark fringes are of equal width.	The width of central bright fringe is twice the width of any secondary maximum.
3.	All bright fringes are of same intensity.	Intensity of bright fringes decreases as we move away from central bright fringe on either side.
4.	Regions of dark fringes are perfectly dark. So there is a good contrast between bright and dark fringes.	Regions of dark fringes are not perfectly dark. So there is a poor contrast between bright and dark fringes.
5.	At an angle of λ/d, we get a bright fringe in the interference pattern of two narrow slits separated by a distance d.	At an angle of λ/a, we get the first dark fringe in the diffraction pattern of a single slit of width a.

10.26 DIFFRACTION AS A LIMIT ON RESOLVING POWER

29. *How does diffraction limit the resolving power of an optical instrument ? Define the terms limit of resolution and resolving power.*

Diffraction as a limit on resolving power. All optical instruments like lens, telescope, microscope etc., act as apertures. Light on passing through them undergoes diffraction. This puts the limit on their resolving power. Suppose a lens is used to form the image of an object. We can think of the lens as a circular aperture. The image of each point is a set of alternate bright and dark circular fringes with a bright disc at the centre. The size of this disc depends on the aperture of the lens and the wavelength of light used. If we have two nearby point objects, their images may give rise to diffraction patterns which overlap on each other, making the identification or resolution of the two objects difficult.

Limit of resolution. *The smallest linear or angular separation between two point objects at which they can be just separately seen or resolved by an optical instrument is called the limit of resolution of the instrument.*

Resolving power. *The resolving power of an optical instrument is its ability to resolve or separate the images of two nearby point objects so that they can be distinctly seen. It is equal to the reciprocal of the limit of resolution of the optical instrument.*

The smaller the limit of resolution of an optical instrument, greater is its resolving power.

30. *State Rayleigh's criterion for resolution of two objects.*

Rayleigh's criterion for resolution. If we look through a telescope at two stars lying closed together, their different patterns overlap and the resultant pattern is little broader but otherwise similar to that of a single star, as shown in Fig. 10.29. So the two stars cannot be resolved or separately seen.

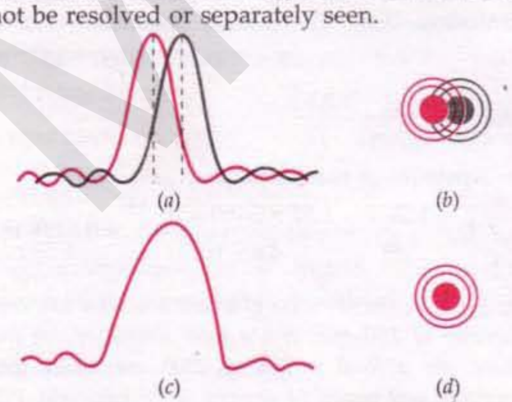


Fig. 10.29 (*a*), (*b*) Overlapping of diffraction patterns of two close point objects. (*c*), (*d*) their resultant diffraction patterns.

According to *Rayleigh's criterion, the images of two point objects are just resolved when the central maximum of the diffraction pattern of one falls over the first minimum of the diffraction pattern of the other,* as shown in Fig. 10.30. When seen through the telescope, the resultant diffraction pattern has a well-marked depression at the top, showing that these are really two stars and not one. Thus the images of two stars have been just resolved.

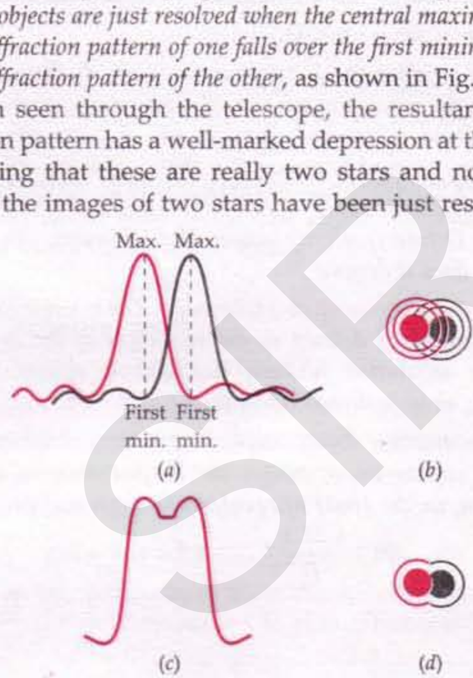


Fig. 10.30 Rayleigh's criterion for resolution.

10.27 RESOLVING POWER OF MICROSCOPE AND TELESCOPE

31. *Define resolving power of microscope. Write an expression for it. What is the advantage of using oil immersion objective.*

Resolving power of a microscope. *The resolving power of a microscope is defined as reciprocal of the smallest distance between two point objects at which they can be just resolved when seen through the microscope.*

The smallest distance between two point objects at which they can be just resolved by the microscope, or the ***limit of resolution***, is given by

$$d = \frac{\lambda}{2\mu \sin \theta}$$

$$\therefore \text{ Resolving power of a microscope} = \frac{1}{d} = \frac{2\mu \sin \theta}{\lambda}$$

Here,

λ = the wavelength of light used,

θ = half the angle of cone of light from each point object or the angle subtended by each point object on the radius of the objective[Fig. 10.31].

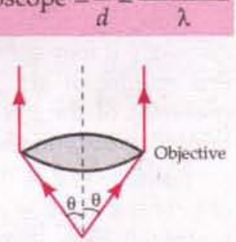


Fig. 10.31

μ = the refractive index of the medium between the point object and the objective of the microscope.

The factor $\mu \sin\theta$ is called the ***numerical aperture*** (for eye, $\mu \sin\theta = 0.004$).

The smaller the limit of resolution '*d*', the greater will be the resolving power. The resolving power of a microscope increases when an oil of high refractive index (μ) is used between the object and the objective (called the ***oil immersion objective***) of the microscope.

32. *Define resolving power of a telescope. On what factors does it depend ?*

Resolving power of a telescope. *The resolving power of a telescope is defined as the reciprocal of the smallest angular separation between two distant objects whose images can be just resolved by it.*

The *smallest linear angular separation between two distant objects whose images can be just resolved by the telescope*, or the ***limit of resolution***, is given by

$$d\theta = \frac{1.22\,\lambda}{D}$$

$$\therefore \text{ Resolving power of a telescope} = \frac{1}{d\theta} = \frac{D}{1.22\,\lambda}$$

Here

λ = the wavelength of light,

D = the diameter of the telescope objective, and

$d\theta$ = the angle subtended by the two distant objects at the objective.

Thus larger the aperture of the objective and smaller the wavelength of light used, the greater will be the resolving power of the telescope.

33. *Give an estimation of the resolving power of the human eye.*

Resolving power of human eye. The diameter of the pupil of human eye is about 2 mm. If we take $\lambda = 5000$ Å, then the smallest angular separation between two distant point objects that the human eye can resolve will be

$$d\theta = \frac{1.22\,\lambda}{D} = \frac{1.22 \times 5000 \times 10^{-10}}{2 \times 10^{-3}} = 0.305 \times 10^{-3} \text{ rad} \approx 1'$$

Thus, the human eye can see two point objects distinctly if they subtend, at the eye, an angle equal to one minute of arc. This angle is called the *limit of resolution* of the eye. The reciprocal of this angle or limit of resolution gives the resolving power of the eye.

Further, if d is the separation between two point objects at a distance of 1 km which can be just resolved by human eyes, then

$$0.305 \times 10^{-3} = \frac{d}{10^3} \quad \text{or} \quad d = 0.305 \text{ m} = 30.5 \text{ cm}$$

Thus the human eyes can see two objects separated by 30 cm just resolved from a distance of 1 km.

Examples based on

Resolving Power of (i) Telescope (ii) Microscope

Formulae Used

1. Limit of resolution of a telescope, $d\theta = \frac{1.22\,\lambda}{D}$.
2. Resolving power of a telescope $= \frac{1}{d\theta} = \frac{D}{1.22\,\lambda}$

   where D = diameter of the objective lens.
3. Limit of resolution of a microscope, $d = \frac{\lambda}{2\mu \sin\theta}$
4. Resolving power of a microscope $= \frac{1}{d} = \frac{2\mu \sin\theta}{\lambda}$

   where θ = half angle of cone of light from the point object. The factor $\mu \sin\theta$ is called numerical aperture (N.A.).

Units Used

Lengths λ, D and d are in metre while angles θ and $d\theta$ are in radian.

Example 62. *Assume that light of wavelength 6000 Å is coming from a star. What is the limit of resolution of a telescope whose objective has a diameter of 100 inch ?* [NCERT]

Solution. The limit of resolution of a telescope,

$$d\theta = \frac{1.22\lambda}{D}$$

Here $D = 100$ inch $= 254$ cm,

$\lambda = 6000$ Å $= 6 \times 10^{-5}$ cm

$$\therefore \quad d\theta = \frac{1.22 \times 6 \times 10^{-5}}{254} = \mathbf{2.9 \times 10^{-7}\ rad.}$$

Example 63. *A telescope is used to resolve two stars separated by 4.6×10^{-6} rad. If the wavelength of light used is 5460 Å, what should be the aperature of the objective of the telescope ?*

Solution. Here $d\theta = 4.6 \times 10^{-6}$ rad,

$\lambda = 5460$ Å $= 5460 \times 10^{-10}$ m

As $$d\theta = \frac{1.22\,\lambda}{D}$$

$\therefore$ Aperture of the telescope,

$$D = \frac{1.22\,\lambda}{d\theta} = \frac{1.22 \times 5460 \times 10^{-10}}{4.6 \times 10^{-6}} = \mathbf{0.1488\ m.}$$

Example 64. *The objective of an astronomical telescope has a diameter of 150 mm and a focal length of 4.0 m. The eyepiece has a focal length of 25.0 mm. Calculate the magnifying and resolving powers of the telescope. What is the distance between the objective and the eyepiece ?*

Take $\lambda = 6000$ Å. [CBSE D 93 C]

Solution. Assume that the final image is formed at infinity. Then

Magnifying power,

$$m = \frac{f_0}{f_e} = \frac{4}{25 \times 10^{-3}} = \mathbf{160}$$

Resolving power

$$= \frac{D}{1.22\,\lambda} = \frac{150 \times 10^{-3}}{1.22 \times 6000 \times 10^{-10}} = \mathbf{2.049 \times 10^5}.$$

Distance between objective and eyepiece

$$= f_0 + f_e = 4 + 25 \times 10^{-3} = \mathbf{4.025\ m}.$$

Example 65. *Calculate the separation of two points on the moon that can be resolved using 600 cm telescope. Given the distance of the moon from the earth is 3.8×10^{10} cm. The wavelength most sensitive to the eye is 5.5×10^{-5} cm.*

Solution. Here $D = 600$ cm, $\lambda = 5.5 \times 10^{-5}$ cm

Limit of resolution,

$$d\theta = \frac{1.22\,\lambda}{D} = \frac{1.22 \times 5.5 \times 10^{-5}}{600} = 1.1 \times 10^{-7}\ \text{rad}$$

Let x be the distance between two points on the moon and d be the distance between the moon and the objective of the telescope. Then

$$d\theta = \frac{x}{d}$$

or $\quad x = d \times d\theta = 3.8 \times 10^{10} \times 1.1 \times 10^{-7} = \mathbf{4180\ cm}.$

Example 66. *A telescope has an objective of diameter 60 cm. The focal lengths of the objective and eye-piece are 2.0 m and 1.0 cm respectively. The telescope is directed to view two distant almost point sources of light (e.g. two stars of a binary). The sources are at roughly the same distance ($= 10^4$ light years) along the line of sight but are separated transverse to the line of sight by a distance of 10^{10} m. Will the telescope resolve the two objects i.e. will it see two distinct stars ?* [NCERT]

Solution. Separation between the two stars is

$$y = 10^{10}\ \text{m}$$

Distance of the stars from the earth is

$$x = 10^4\ \text{light years} = 10^4 \times 9.47 \times 10^{15}\ \text{m}$$

∴ Angle subtended by the line joining the two stars on the objective lens (or on eye) is

$$d\theta = \frac{y}{x} = \frac{10^{10}}{10^4 \times 9.47 \times 10^{15}} = 0.106 \times 10^{-9}\ \text{rad}$$

Now diameter of objective, $D = 60\ \text{cm} = 0.60\ \text{m}$

For mean yellow colour, $\quad \lambda = 600\ \text{nm} = 6 \times 10^{-7}\ \text{m}$

According to Rayleigh's criterion, the limit of resolution of the telescope is

$$d\theta' = \frac{1.22\,\lambda}{D} = \frac{1.22 \times 6 \times 10^{-7}}{0.60} = \mathbf{1.22 \times 10^{-6}\ rad}$$

As the angle $d\theta$ subtended by the transverse separation of the two stars is much too small compared to the limit of resolution $d\theta'$, hence the two stars of the binary cannot be resolved by the given telescope.

Example 67. *Calculate the resolving power of a microscope if its numerical aperture is 0.12 and the wavelength of light used is 6000 Å.*

Solution. Here

$$\text{N.A.} = 0.12, \quad \lambda = 6000\ \text{Å} = 6 \times 10^{-7}\ \text{m}$$

R.P. of the microscope

$$= \frac{2 \times \text{N.A.}}{\lambda} = \frac{2 \times 0.12}{6 \times 10^{-7}} = \mathbf{4 \times 10^5\ m^{-1}}.$$

Example 68. *Calculate the numerical aperture of a microscope required to just resolve two points separated by a distance of 10^{-4} cm, using light of wavelength 5.8×10^{-5} cm.*

Solution. Here $\lambda = 5.8 \times 10^{-5}$ cm, $d = 10^{-4}$ cm

As $\quad d = \dfrac{\lambda}{2 \times \text{N.A.}}$

$$\text{N.A.} = \frac{\lambda}{2\,d} = \frac{5.8 \times 10^{-5}}{2 \times 10^{-4}} = \mathbf{0.29}.$$

Example 69. *The smallest object detail that can be resolved with a certain microscope with light of wavelength 6000 Å is 3.5×10^{-5} cm. Find the numerical aperture of the objective (i) when used dry and (ii) when immersed in an oil of refractive index 1.5.*

Solution. Here $\lambda = 6000\ \text{Å} = 6 \times 10^{-7}$ m,

$$d = 3.5 \times 10^{-5}\ \text{cm} = 3.5 \times 10^{-7}\ \text{m}$$

(*i*) When the objective is used dry,

$$\text{N.A.} = \frac{\lambda}{2d} = \frac{6 \times 10^{-7}}{2 \times 3.5 \times 10^{-7}} = \mathbf{0.86}.$$

(*ii*) When the objective is immersed in an oil of refractive index 1.5,

$$\text{N.A.} = \mu \times \text{dry aperture} = 1.5 \times 0.86 = \mathbf{1.44}.$$

Example 70. *Assuming the diameter of the eye pupil to be 2.0 mm, calculate the smallest angular separation at which two point objects can be distinctly seen when viewed in light of wavelength 6000 Å.* **[CBSE D 91]**

Solution. Here $\lambda = 6000\ \text{Å} = 6 \times 10^{-7}$ m,

$$d = 2.0\ \text{mm} = 2.0 \times 10^{-3}\ \text{m}$$

The limit of resolution of the eye,

$$d\theta = \frac{1.22\,\lambda}{d}$$

$$= \frac{1.22 \times 6 \times 10^{-7}}{2.0 \times 10^{-3}} = \mathbf{3.66 \times 10^{-4}\ rad}.$$

Problems For Practice

1. Roughly calculate the limit of resolution of 100 cm telescope with visible light of wavelength $\lambda = 5500$ Å. (Ans. 6.71×10^{-7} rad)
2. Calculate the resolving power of this telescope, assuming the diameter of the objective lens to be 6 cm and the wavelength of light used to be 540 nm. **[CBSE D 06C]** (Ans. 9.1×10^4)
3. The objective of a telescope has a diameter of 125 cm. Calculate the smallest angular separation of two stars, measured in seconds, that may be resolved by it. Mean wavelength of light = 6000 Å. (Ans. 0.12 second of arc)
4. What is the aperture of the objective of a telescope that can be used to just resolve stars separated by 6×10^{-6} rad. Given $\lambda = 5.8 \times 10^{-5}$ cm. (Ans. 11.79 cm)
5. Two point objects, separated by a distance of 6×10^{-5} cm, are to be resolved using a microscope. Calculate the numerical aperture if light of wavelength 546 nm is used. (Ans. 4.55×10^{-3})
6. Calculate the limit of resolution of a microscope if an object of numerical aperture 0.12 is viewed by using light of wavelength 6×10^{-7} m. (Ans. 2.55×10^{-8} cm)

HINTS

1. Limit of resolution of a telescope,
$$d\theta = \frac{1.22\,\lambda}{D}$$
Here $D = 100$ cm, $\lambda = 5500$ Å $= 5500 \times 10^{-8}$ cm
$$\therefore \quad d\theta = \frac{1.22 \times 5500 \times 10^{-8}}{100}\text{ rad} = \mathbf{67.1 \times 10^{-8}\ rad.}$$
2. R.P. of the telescope
$$= \frac{1}{d\theta} = \frac{D}{1.22\lambda} = \frac{6 \times 10^{-2}}{1.22 \times 540 \times 10^{-9}} = \mathbf{9.1 \times 10^4}.$$
3. $$d\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 6000 \times 10^{-10}}{125 \times 10^{-2}}$$
$$= 5.85 \times 10^{-7}\text{ rad}$$
$$= \mathbf{0.12\ second\ of\ arc}$$
4. Use $D = \dfrac{1.22\,\lambda}{d\theta}$.
5. Use N.A. $= \dfrac{\lambda}{2d}$.
6. Limit of resolution of a microscope $= \dfrac{\lambda}{2 \times \text{N.A.}}$.

10.28 POLARISATION OF WAVES : INTRODUCTION

34. *What are unpolarised and polarised waves ? Explain polarisation, taking an example of mechanical waves. Can longitudinal waves be polarised ?*

Polarisation of waves. The waves are of *two* types : *Transverse* and *Longitudinal*. Both types of these waves undergo reflection, refraction, interference and diffraction. The difference is that only transverse waves can be polarised.

A transverse wave in which vibrations are present in all possible directions, in a plane perpendicular to the direction of propagation, is said to be ***unpolarised.*** *If the vibrations of a wave are present in just one direction in a plane perpendicular to the direction of propagation, the wave is said to be* ***polarised*** *or* ***plane polarised.*** *The phenomenon of restricting the oscillations of a wave to just one direction in the transverse plane is called* ***polarisation of waves.***

Experimental demonstration with mechanical waves. Consider a long string AB passing through two rectangular slits S_1 and S_2, as shown in Fig. 10.32. The end B of the string is tied to a hook in a wall and the free end A is jerked in all possible directions perpendicular to the length of the string so as to generate transverse waves in it. The portion AS_1 of the string has vibrations in all directions perpendicular to AB, so that the wave is unpolarised. The first slit S_1 will permit only those vibrations to pass through it which are parallel to the slit S_1 and will cut off all other vibrations. Thus the wave emerging from the slit S_1 is plane polarised. The slit S_1 is called the *polariser*. If the second slit S_2, called the *analyser*, is held parallel to S_1, the wave from S_1 will pass through S_2 unchanged. If S_2 is held perpendicular to S_1, no vibrations will emerge from the slit S_2. This indicates that the slit S_1 has polarised the incoming wave in the vertical plane.

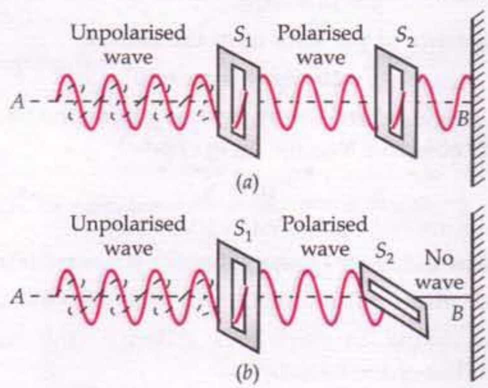


Fig. 10.32 Experiment to demonstrate polarisation of a wave on a string.

Longitudinal waves cannot be polarised. This is because these waves are *symmetrical* about the direction of propagation. For example, if we pass a long spring through two slits and generate a longitudinal wave in it by alternately compressing and releasing its free end, it is seen that the compressions and rarefactions pass through the two slits, whatever is their relative orientation. This is so because the oscillations occur along the length of the spring, *i.e.*, along the direction of the wave propagation. On the other hand, the transverse waves can be polarised as they do not show any symmetry about the direction of wave propagation.

10.29 UNPOLARISED AND PLANE POLARISED LIGHT

35. *What do you mean by unpolarised and plane polarised lights ? Give their pictorial representations.*

Unpolarised light. In ordinary light, electric field vector vibrates in all directions in a plane perpendicular to the direction of propagation. *A light which has vibrations in all directions in a plane perpendicular to the direction of propagation is said to be* **unpolarised light.** The light from the sun, a sodium lamp, an incandescent bulb or a candle is unpolarised. The electric field vector of such a light takes all possible directions in the transverse plane, rapidly and randomly, during the time of measurement, as shown in Fig. 10.33(*a*). The tip of the electric field vector traces an irregular curve, as shown in Fig. 10.33(*b*).

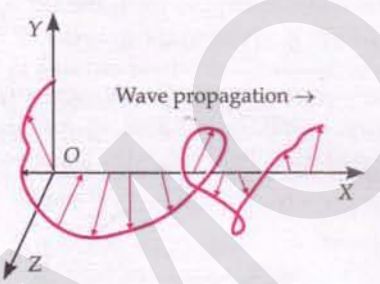


Fig. 10.33 (*a*) Random variation of the electric field vector of unpolarised light.

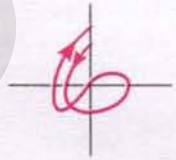


Fig. 10.33 (*b*) Irregular curve traced by the tip of this vector.

Figure 10.34(*a*) is the pictorial representation of unpolarised light propagating out of the plane of paper. It shows vibrations in all directions in the transverse plane. Figure 10.34(*b*) is also a pictorial representation of an unpolarised light. Here double arrows represent the vibrations in the plane of paper and small dots represent vibrations perpendicular to the plane of paper.

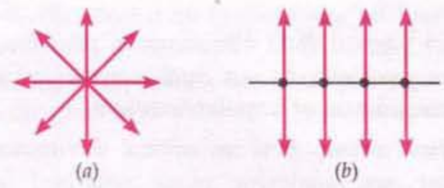


Fig. 10.34 Pictorial representations of unpolarised light.

Plane polarised or linearly polarised light. *If the electric field vector of a light wave vibrates just in one direction perpendicular to the direction of wave propagation, then it is said to be* ***linearly polarised***. Since in a linearly polarised wave, the vibrations at all points, at all times, lie in the same plane, so it is also called a ***plane polarised wave***. Fig. 10.35(*a*) shows the regular variation of the electric field vector of a linearly polarised light along *Y*-axis. Its tip vibrates back and forth along a straight line [Fig. 10.35(*b*)].

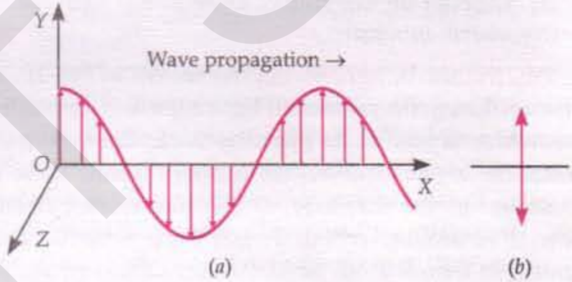


Fig. 10.35(*a*) Regular variation of electric field vector of linearly polarised light. (*b*) The straight path traced by the tip of this vector.

Figs. 10.36 (*a*) and (*b*) show the pictorial representations for polarised right.

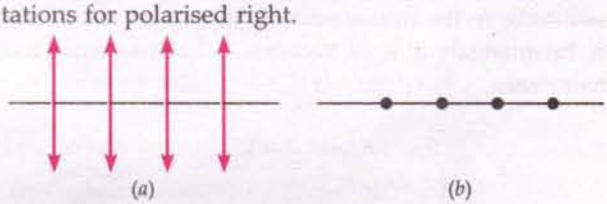


Fig. 10.36 Plane polarised light :
(*a*) Vibrations parallel to the plane of paper,
(*b*) Vibrations perpendicular to the plane of paper.

10.30 POLARISERS

36. *What are polarisers ? Name some commonly used polarisers.*

Polarisers. *A device that plane-polarises the unpolarised light passed through it is called a polariser.*

Some commonly used polarisers are as follows :

1. ***Tourmaline crystal.*** The tourmaline crystal is so cut that its plane contains its optic axis. When

unpolarised light is incident on it normally, it allows only such electric field vibrations to pass through it which are parallel to its axis. Such a crystal can be used to polarise a beam of unpolarised light.

2. ***Nicol prism.*** It is an optical device used for producing and analysing plane polarised light. It consists of two pieces of calcite suitably cut and stuck together with Canada balsam.

3. ***Polaroid.*** A polaroid is a thin commercial sheet in the form of circular disc which makes use of the property of selective absorption to produce an intense beam of plane polarised light.

We shall study nicol prisms and polaroids in detail within next few sections.

10.31 EXPERIMENT TO DEMONSTRATE POLARISATION OF LIGHT

37. *Describe an experiment to show that light waves are transverse in nature.*

Polarisation of light waves. As shown in Fig. 10.37, when ordinary (unpolarised) light is passed through a tourmaline crystal *P*, its intensity is cut down to half. Rotate the crystal *P* about the incident beam. There is no effect on the intensity of the transmitted beam. Now, if a similar crystal *A* is placed with its axis parallel to that of *P*, all the light transmitted by *P* also passes through *A*. However, as *A* is rotated from this position in its own plane, the intensity of light transmitted by it goes on decreasing until it becomes zero, when the axes of the two crystals are perpendicular to each other. The two crystals are then said to be in the *crossed position*. In one full rotation of *A*, the intensity of light becomes twice maximum and twice zero.

Clearly, the light transmitted by crystal *P* is polarised as it contains vibrations parallel to the axis of *P*. When the axis of *A* is parallel to that of *P*, the polarised beam passes as such through *A* also. But when the axis of *A* is perpendicular to that of *P*, the vibrations of the waves (being perpendicular to the axis of *A*) are not transmitted by *A*. The first crystal *P* which polarises the light is called *polariser* and the second crystal is called the *analyser*, because it analyses whether the light is polarised or not.

Since the polarisation of light can neither be explained on the basis of corpuscular theory of light nor by assuming that light propagates as longitudinal waves, therefore, *the above experiment proves that light propagates in the form of transverse waves.*

10.32 LAW OF MALUS

38. *State and explain the law of Malus. Sketch a graph showing the dependence of intensity of transmitted light on the angle between polariser and analyser.*

Law of Malus. When a plane polarised light is seen through an analyser, the intensity of transmitted light varies as the analyser is rotated in its own plane about the incident direction. In 1809, *E.N. Malus* discovered that *when a beam of completely plane polarised light is passed through analyser, the intensity 'I' of transmitted light varies directly as the square of the cosine of the angle 'θ' between the transmission directions of polariser and analyser.* This statement is known as the ***law of Malus.***

Mathematically,

$$I \propto \cos^2 \theta \quad \text{or} \quad I = I_0 \cos^2 \theta$$

Here I_0 is the maximum intensity of transmitted light. It may be noted that I_0 is equal to half the intensity of unpolarised light incident on the polariser.

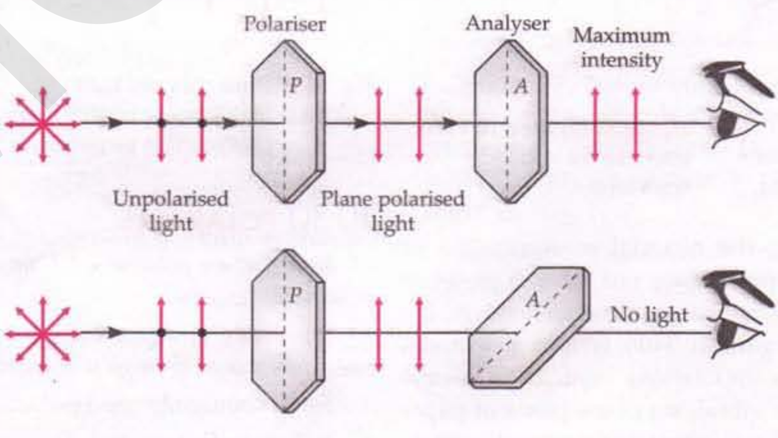


Fig. 10.37 Demonstration of polarisation of light.

Explanation of the law. As shown in Fig. 10.38, suppose that the planes of polariser and analyser are inclined to each other at an angle θ. Let I_0 be the intensity and a the amplitude of the plane polarised light transmitted by the polariser.

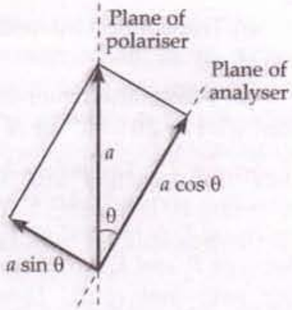


Fig. 10.38 Law of Malus.

The amplitude a of the light incident on the analyser has *two* rectangular components :

1. $a \cos\theta$, parallel to the plane of transmission of the analyser, and
2. $a \sin\theta$, perpendicular to the plane of transmission of the analyser.

So only the component $a\cos\theta$ is transmitted by the analyser. The intensity of light transmitted by the analyser is

$$I = k(a\cos\theta)^2 = ka^2\cos^2\theta$$

or $$I = I_0\cos^2\theta$$

where $I_0 = ka^2$, is the maximum intensity of light transmitted by the analyser (when $\theta = 0°$). The above equation is the ***law of Malus***.

Special Cases

1. When $\theta = 0°$ or $180°$, $\cos\theta = \pm 1$, so that $I = I_0$
   So *when the transmission directions of polariser and analyser are parallel or antiparallel to each other, the maximum intensity of plane polarised light is transmitted by the analyser and is equal to the intensity emerging from the polariser.*
2. When $\theta = 90°$, $\cos\theta = 0$, so that $I = 0$
   So *when the transmission directions of polariser and analyser are perpendicular to each other, the intensity of light transmitted through the analyser is zero.*
3. When a beam of unpolarised light is incident on the polariser,

$$I = I_0\overline{\cos^2\theta} = I_0 \times \frac{1}{2}\overline{(1+\cos 2\theta)}$$

$$= \frac{1}{2}I_0(1+\overline{\cos 2\theta}) = \frac{1}{2}I_0(1+0) = \frac{1}{2}I_0.$$

Intensity curve. As the angle 'θ' between the transmission directions of polariser and analyser is varied, the intensity 'I' of the light transmitted by the analyser varies as a function of $\cos^2\theta$, as shown in Fig. 10.39.

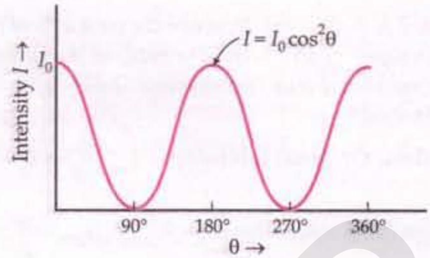


Fig. 10.39 Graph of intensity I through analyser versus angle θ between polariser and analyser.

Examples based on The Law of Malus

Formula Used

Law of Malus, $I = I_0\cos^2\theta$.

Units Used

Intensities I and I_0 are in watt / m^2 and angle θ in degrees.

Example 71. *Two polaroids are used to study polarisation. One of them (the polariser) is kept fixed and the other (the analyser) is initially kept with its axis parallel to the polariser axis. The analyser is then rotated through angles of 45°, 90° and 180° in turn. How would the intensity of light coming out of the analyser be affected for these angles of rotation, as compared to the initial intensity and why ?* [CBSE D 03C]

Solution. Let I_0 be the intensity when the axis of analyser is parallel to the axis of polariser. By the law of Malus,

$$I = I_0\cos^2\theta$$

(*i*) When $\theta = 45°$, $I = I_0\cos^2 45° = \frac{I_0}{2}$.

(*ii*) When $\theta = 90°$, $I = I_0\cos^2 90° = \mathbf{0}$.

(*iii*) When $\theta = 180°$, $I = I_0\cos^2 180° = I_0$.

Example 72. *Two polarising sheets have their polarising directions parallel so that the intensity of the transmitted light is maximum. Through what angle must the either sheet be turned if the intensity is to drop by one-half ?*

Solution. Here $I = \frac{I_0}{2}$

Using Malus law, $I = I_0\cos^2\theta$

$\therefore \quad \frac{I_0}{2} = I_0\cos^2\theta \quad$ or $\quad \cos\theta = \pm\frac{1}{\sqrt{2}}$

Hence $\theta = \pm 45°, \pm 135°$

The same effect occurs no matter which sheet is rotated or in which direction it is rotated.

Example 73. *If the angle between the pass axis of polarizer and the analyser is 45°, write the ratio of the intensities of original light and the transmitted light after passing through the analyser.* [CBSE D 09]

Solution. Original intensity, $I_{\text{original}} = I_0$

Transmitted intensity, $I_{\text{transmitted}} = \frac{I_0}{2}\cos^2\theta$

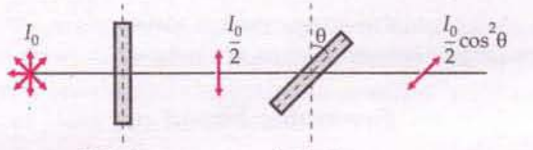


Fig. 10.40

Given $\theta = 45°$,

$$\therefore \frac{I_{\text{original}}}{I_{\text{transmitted}}} = \frac{I_0}{\frac{I_0}{2}\cos^2 45°} = \frac{1}{\frac{1}{2}\times\frac{1}{2}} = \frac{4}{1} = \mathbf{4:1.}$$

Example 74. *The polaroids P_1 and P_2 are placed in crossed positions. A third polaroid P_3 is kept between P_1 and P_2 such that pass axis of P_3 is parallel to that of P_1. How would the intensity of light (I_2) transmitted through P_2 vary as P_3 is rotated ? Draw a plot of intensity 'I_2' vs the angle 'θ' between pass axes of P_1 and P_3.*

In which orientation will the transmitted intensity be (i) minimum and (ii) maximum ? [CBSE D 15 ; OD 10, 15C]

Solution.

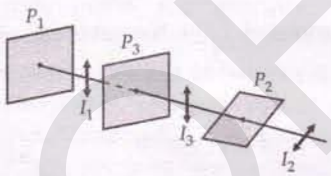


Fig. 10.41 (*a*)

As θ is the angle between P_1 and P_3, the angle between P_3 and P_2 would be $(90° - \theta)$.

By Malus law, $I_3 = I_1\cos^2\theta$

and $\quad I_2 = I_3\cos^2(90° - \theta) = I_3\sin^2\theta$

or $\quad I_2 = I_1\cos^2\theta\sin^2\theta = \frac{1}{4}I_1(2\sin\theta\cos\theta)^2$

or $\quad I_2 = \frac{1}{4}I_1\sin^2 2\theta$

The plot of I_2 *vs*. θ will be of the form as shown in Fig. 10.41(*b*)

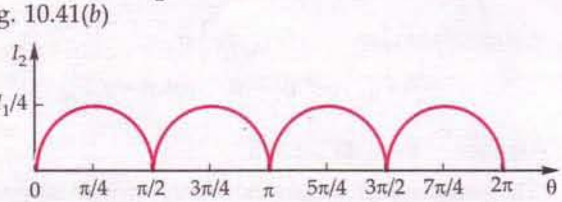


Fig. 10.41 (*b*)

(*i*) Transmitted intensity I_2 will be minimum when $\sin 2\theta = 0$ or $\theta = 0°$.

(*ii*) Transmitted intensity I_2 will be maximum when $\sin 2\theta = 1$ or $2\theta = 90°$ or $\theta = 45°$.

Example 75. *Two polaroids P_1 and P_2 are placed with their pass axes perpendicular to each other. Unpolarised light of intensity I_0 is incident on P_1. A third polaroid P_3 is kept in between P_1 and P_2 such that its pass axis makes an angle of 60° with that of P_1. Determine the intensity of light transmitted through P_1, P_2 and P_3.* **[CBSE OD 14]**

Solution. Intensity of light through $P_1 = \frac{I_0}{2}$

Intensity of light through P_3

$$= \frac{I_0}{2}\cos^2 60° = \frac{I_0}{2}\cdot\left(\frac{1}{2}\right)^2 = \frac{I_0}{8}$$

Intensity of light through P_2

$$= \frac{I_0}{8}\cdot\cos^2(90° - 60°) = \frac{I_0}{8}\cdot\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{32}I_0.$$

Example 76. *Unpolarised light of intensity I_0 passes through two polaroids P_1 and P_2 such that pass axis of P_2 makes an angle θ with the pass axis of P_1. A third polaroid P_3 is placed between P_1 and P_2 with pass axis of P_3 making an angle β with that of P_1. If I_1, I_2 and I_3 represent the intensities of light transmitted by P_1, P_2 and P_3, determine the values of angle θ and β for which $I_1 = I_2 = I_3$.* [CBSE OD 14C]

Solution.

I_1 = Light transmitted by P_1

I_3 = Light transmitted by $P_3 = I_1\cos^2\beta$

I_2 = Light transmitted by $P_2 = I_3\cos^2(\theta - \beta)$

As $\quad I_2 = I_3 \Rightarrow \quad I_3\cos^2(\theta - \beta) = I_3$

$\therefore \cos^2(\theta - \beta) = 1 \quad \Rightarrow \cos(\theta - \beta) = 1 = \cos 0 \Rightarrow \theta = \beta$

Also, $\quad I_1 = I_2 = I_1\cos^2\beta\cos^2(\theta - \beta)$

or $\quad 1 = \cos^2\theta\cos^2 0 \quad \Rightarrow \cos\theta = 1 = \cos 0$

Hence, $\theta = \beta = \mathbf{0}$ or π **rad**

Example 77. *Two polaroids are placed 90° to each other. What happens when $N - 1$ more polaroids are inserted between two crossed polaroids (at 90° to each other). Their axes are equally spaced. How does the transmitted intensity behave for large N ?* [NCERT]

Solution. Transmitted intensity through first polaroid is

$$I_1 = I_0\cos^2\theta$$

where I_0 is the original intensity. Similarly, the transmitted intensity through second polaroid will be

$$I_2 = I_1\cos^2\theta = I_0\cos^4\theta$$

If N polaroids are used, then $\quad I_N = I_0(\cos\theta)^{2N}$

As the optic axes of the polaroids are equally inclined, so angle of rotation θ is same for each polaroid.

Thus $\quad \frac{I_N}{I_0} = (\cos\theta)^{2N}$

But angle between successive polaroids is

$$\theta = \frac{90°}{N} = \frac{\pi}{2N} \text{ radians}$$

$$\therefore \left(\cos\frac{\pi}{2N}\right)^{2N} = \left(1 - \frac{\pi^2}{8N^2} + \ldots\right)^{2N} \simeq \left[1 - \frac{2N\pi^2}{8N^2} + \ldots\right]$$

which approaches 1 for large N. Hence fractional intensity,

$$\frac{I_N}{I_0} = 1 \quad \text{or} \quad I_N = I_0$$

Example 78. *A polaroid examines two adjacent plane-polarised light beams A and B whose planes of polarisation are mutually at right angles. In one position of the polaroid, the beam B shows zero intensity. From this position a rotation of 30° shows the two beams of equal intensities. Find the intensity ratio I_A / I_B of the two beams.*

Solution. The planes of polarisation of light beams A and B are mutually at right angles. Initially, the beam B shows zero intensity. Therefore, $\theta = 90°$ for beam B and $\theta = 0°$ for beam A. When the polaroid is rotated through 30°, we have $\theta = 60°$, for beam A and $\theta = 30°$ for beam B. In this position,

Intensity of emerging beam A
= Intensity of emerging beam B

$$\therefore \quad I_A \cos^2 30° = I_B \cos^2 60°$$

or $$\frac{I_A}{I_B} = \frac{\cos^2 60°}{\cos^2 30°} = \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{1}{3} = \mathbf{1:3}$$

Problems For Practice

1. Two polaroids P_1 and P_2 are placed with their pass axes perpendicular to each other. Unpolarised light of intensity I_0 is incident on P_1. A third polaroid P_3 is kept in between P_1 and P_2 such that its pass axis makes an angle of 45° with that of P_1. Determine the intensity of light transmitted through P_1, P_2 and P_3. **[CBSE OD 14]** (Ans. $I_0/2$, $I_0/4$, $I_0/8$)
2. A polariser and an analyser are oriented so that the maximum light is transmitted. What is the fraction of maximum light transmitted when analyser is rotated through (*i*) 30° (*ii*) 60° ? [Ans. (*i*) 0.75, (*ii*) 0.25]
3. Unpolarised light falls on two polarising sheets placed one on the top of the other. What must be the angle between the characteristic directions of the sheets if the intensity of the transmitted light is (*a*) one-third of the maximum intensity of the transmitted beam (*b*) one third of the intensity of the incident beam. (Ans. ± 55°, ± 35°)
4. Two polaroids are crossed to each other. If one of them is rotated through 60°, then what percentage of the incident unpolarised light will be transmitted by the polaroids ? (Ans. 37.5%)
5. Four polaroids are so placed that the transmission-axis of each is inclined at an angle of 30° from the axis of the previous polaroid in the same direction. If unpolarised light-beam of intensity I_0 falls on the first polaroid, then what will be the intensity of the light emerging from the last polaroid ? (Ans. 0.21 I_0)

HINTS

1. Proceed as in Example 75.
2. (*i*) $I = I_0 \cos^2 30° = I_0 \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} I_0$ or $\frac{I}{I_0} = \mathbf{0.75}$.

   (*ii*) $I = I_0 \cos^2 60° = I_0 \left(\frac{1}{2}\right)^2 = \frac{I_0}{4}$ or $\frac{I}{I_0} = \mathbf{0.25}$.
3. (*i*) $\frac{I_0}{3} = I_0 \cos^2\theta$ (*ii*) $\frac{2I_0}{3} = I_0 \cos^2\theta$
4. Intensity of incident unpolarised light = I_0

   Intensity transmitted by first polaroid, $I_1 = \frac{I_0}{2}$

   Intensity transmitted by second polaroid,

   $$I_2 = I_1 \cos^2(90° - 60°) = \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8} I_0 = 0.375\, I_0$$

   Transmitted percentage

   $$= \frac{I_2}{I_0} \times 100 = 0.375 \times 100 = \mathbf{37.5\%}$$
5. $I = \frac{I_0}{2} \times (\cos^2\theta)^3 = \frac{I_0}{2} \times (\cos 30°)^6 = \frac{27}{128} I_0 = \mathbf{0.21\, I_0}$

10.33 PLANES OF POLARISATION AND VIBRATION

39. *With the help of a diagram, explain the plane of vibration and plane of polarisation.*

Plane of vibration and plane of polarisation. When ordinary light is passed through a tourmaline crystal, the light is plane polarised and the vibrations of the electric field vector take place just in one direction perpendicular to the direction of propagation of light.

*The plane containing the direction of vibration and the direction of wave propagation is called the **plane of vibration**.*

As shown in Fig. 10.42, if light wave is propagating along *X-axis* and the electric field vector vibrates parallel to Y-axis, then $ABCD$ or XY-plane is the plane of vibration.

*The plane passing through the direction of wave propagation and perpendicular to the plane of vibration is called the **plane of polarisation.** No vibrations occur in the plane of polarisation.*

In Fig. 10.42, *EFGH* or *XZ*-plane is the plane of polarisation.

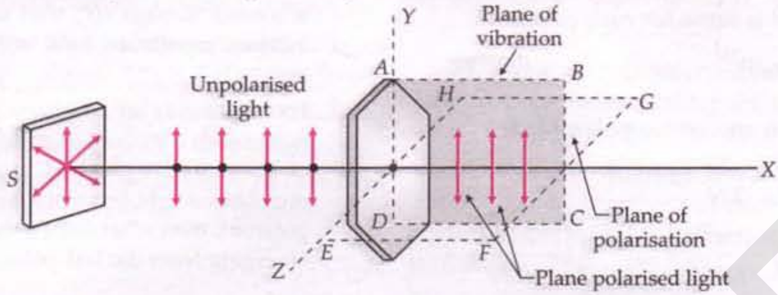


Fig. 10.42 Plane of vibration and plane of polarisation.

10.34 CIRCULARLY AND ELLIPTICALLY POLARISED LIGHTS

40. *What do you understand by circularly and elliptically polarised lights ?*

Circularly polarised light. *If the tip of the electric field vector of a light wave traces a circle, the light is said to be circularly polarised.* It can be regarded as the combination of two plane polarised vibrations of equal amplitudes in two mutually perpendicular directions with a phase difference of $\pi/2$.

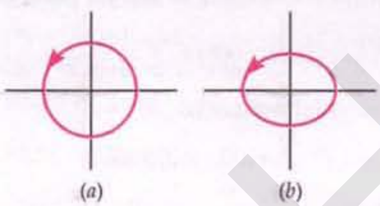


Fig. 10.43 Paths traced by the electric field vectors of
(*a*) circularly polarised and
(*b*) elliptically polarised lights.

Elliptically Polarised light. *If the tip of the electric field vector of a light wave traces an ellipse, the light is said to be elliptically polarised.* It can be regarded as the combination of two plane polarised vibrations of unequal amplitudes in the mutually perpendicular directions with a phase difference of $\pi/2$.

10.35 METHODS OF PRODUCING PLANE POLARISED LIGHT

41. *Name the various methods by which ordinary light can be plane polarised.*

Methods of producing plane polarised light. Ordinary light can be polarised by using any of the following phenomena :

1. Reflection
2. Scattering
3. Double refraction
4. Selective absorption.

10.36 POLARISATION BY REFLECTION : BREWSTER LAW

42. *Explain polarisation by reflection. State and prove Brewster law of polarisation.*

Polarisation by reflection. In 1808, the French physicist *Malus* discovered that when ordinary light is incident on the surface of a transparent medium, the reflected light is partially plane polarised. The extent of polarisation depends on the angle of incidence. For a particular angle of incidence, the reflected light is found to be completely polarised with its vibrations perpendicular to the plane of incidence.

The angle of incidence at which a beam of unpolarised light falling on a transparent surface is reflected as a beam of completely plane polarised light is called ***polarising or Brewster angle***. It is denoted by i_p.

The British physicist *David Brewster* found that at the polarising angle, the reflected and transmitted rays are perpendicular to each other, as shown in Fig. 10.44. Suppose i_p is the polarising angle of incidence and r_p, the corresponding angle of refraction. Then

$$i_p + r_p = 90°$$

or

$$r_p = 90° - i_p$$

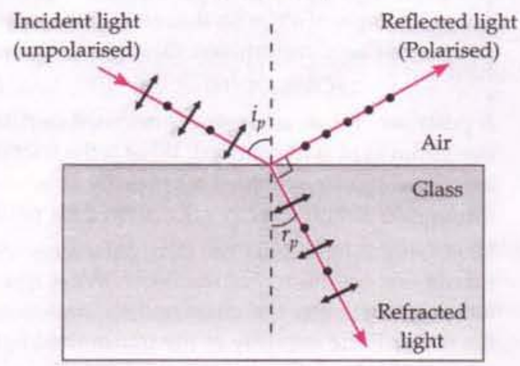


Fig. 10.44 Polarisation of light reflected from a transparent medium at the Brewster angle.

From Snell's law, the refractive index of the transparent medium is

$$\mu = \frac{\sin i_p}{\sin r_p} = \frac{\sin i_p}{\sin(90^\circ - i_p)} = \frac{\sin i_p}{\cos i_p} \quad \text{or} \quad \mu = \tan i_p$$

This relation is known as ***Brewster law.*** The *law states that the tangent of the polarising angle of incidence of a transparent medium is equal to its refractive index.*

The value of Brewster angle depends on the nature of the transparent refracting medium and the wavelength of light used.

Explanation. As shown in Fig. 10.44, the incident unpolarised light has both types of vibrations, one perpendicular (dots) and other parallel to the plane of incidence. At the polarising angle of incidence (i_p), the reflected and refracted rays are perpendicular to each other. The electrons oscillating in the transparent medium produce the reflected wave. These vibrations move in two directions transverse to the refracted wave. As the arrows are parallel to the direction of reflected wave, they cannot send energy along the direction of reflected light. Hence the reflected light consists of vibrations perpendicular to the plane of incidence (dots) only *i.e.,* the reflected light is plane polarised.

Examples based on Brewster Law

Formulae Used

1. Brewster law, $\mu = \tan i_p$ 2. $i_p + r_p = 90^\circ$

3. ${}^2\mu_3 = \frac{{}^1\mu_2}{{}^1\mu_3}$.

Units Used

Angle i_p and r_p are in degrees, refractive index μ has no units.

Example 79. *Unpolarized light is incident on a plane glass. What should be the angle of incidence so that the reflected rays are perpendicular to each other ?* [NCERT]

Solution. Here $i + r = 90^\circ$. Therefore,

$$\tan i_p = \mu = 1.5 \qquad \text{[For glass, } \mu = 1.5]$$

or $$i_p = \mathbf{56.3^\circ}.$$

Example 80. *Yellow light is incident on the smooth surface of a block of dense flint glass for which the refractive index is 1.6640. Find the polarising angle. Also find the angle of refraction.*

Solution. Here $\mu = 1.6640$

By Brewster law, $\tan i_p = \mu \quad \therefore \quad \tan i_p = 1.6640$

Hence $i_p = \tan^{-1}(1.6640) = \mathbf{59.0^\circ}$

If r is the angle of refraction, then $i_p + r = 90^\circ$

$$\therefore \qquad r = 90^\circ - i_p = 90^\circ - 59^\circ = \mathbf{31^\circ}.$$

Example 81. *A ray of light strikes a glass plate at an angle of 60°. If the reflected and refracted rays are perpendicular to each other, find the refractive index of glass.*

Solution. Reflected and refracted rays are mutually perpendicular only when the angle of incidence is equal to polarising angle, hence

$$i_p = 60^\circ$$

$\therefore$ Refractive index,

$$\mu = \tan i_p = \tan 60^\circ = \sqrt{3} = \mathbf{1.732}.$$

Example 82. *At what angle of incidence will the light reflected from water ($\mu = 1.3$) be completely polarised ? Does this angle depend on the wavelength of light ?*

Solution. Here $\mu = 1.3$, $i_p = ?$

As $\tan i_p = \mu = 1.3 \quad \therefore \quad i_p = \tan^{-1} 1.3 = \mathbf{53^\circ}$

Yes, this angle depends on the wavelength of light used.

Example 83. *For a given medium, the polarising angle is 60°. What will be the refractive index and the critical angle for this medium ?* [Himachal 96 ; CBSE D 99]

Solution. Here $i_p = 60^\circ$

$$\therefore \qquad \mu = \tan i_p = \tan 60^\circ = \sqrt{3}$$

$$\sin i_c = \frac{1}{\mu} = \frac{1}{\sqrt{3}} = 0.5774$$

$$\therefore \qquad i_c = \sin^{-1}(0.5774) = \mathbf{35^\circ 16'}.$$

Example 84. *The velocity of light in air is 3×10^8 ms^{-1} and that in water is 2.2×10^8 ms^{-1}. Find the polarising angle of incidence.*

Solution. The refractive index of water is given by

$$\mu = \frac{\text{Speed of light in air}}{\text{Speed of light in water}} = \frac{3 \times 10^8}{2.2 \times 10^8} = 1.3636$$

Using Brewster law, $\tan i_p = \mu = 1.3636$

$$\therefore \quad i_p = \tan^{-1}(1.3636) = \mathbf{53.74^\circ}.$$

Example 85. *The refractive index of water is 4/3 and that of glass 3/2. A beam of light travelling in water enters glass. For what angle of incidence the reflected light will be completely plane-polarised ? (tan 48°22′ = 1.125)*

Solution. Here ${}^a\mu_w = \frac{4}{3}$ and ${}^a\mu_g = \frac{3}{2}$

$$\therefore \qquad {}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{3/2}{4/3} = \frac{9}{8} = 1.125$$

For a beam of light travelling from water to glass,

$$\tan i_p = {}^w\mu_g = 1.125$$

$$i_p = \tan^{-1}(1.125) = \mathbf{48^\circ 22'}.$$

Problems For Practice

1. Find the Brewstar angle for air-water surface for yellow light. Refractive index of water for yellow light = 1.33. (Ans. 53°)
2. A ray of light strikes a glass plate at an angle of incidence 57°. If the reflected and refracted rays are perpendicular to each other, what is the refractive index of glass ? (Ans. 1.54)
3. When sunlight is incident on water at an angle of 53°, the reflected light is found to be completely plane-polarised. Determine (*i*) angle of refraction of light and (*ii*) refractive index of water. [Ans. (*i*) 37° (*ii*) 1.327]
4. In Fig. 10.45, at what angle θ above the horizon should the sun be situated so that its light reflected from the surface of still water of the pond be totally polarised ? Given : refractive index of water $\mu = 1.327$ and $\tan 53° = 1.327$. (Ans. 37°)

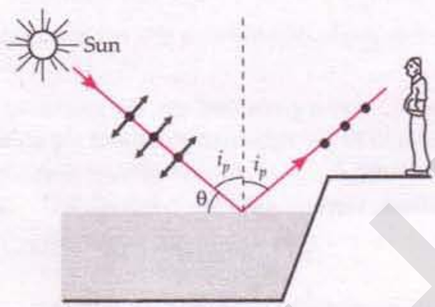


Fig. 10.45

5. The polarising angle for a medium is 60°. Determine (*i*) the refractive index of the medium and (*ii*) the refracting angle. [Ans. (*i*) 1.732 (*ii*) 30°]
6. A ray of light is incident on a glass plate of refractive index 1.54. If the reflected ray is completely plane polarised, find (*i*) angle of incidence (*ii*) angle of refraction and (*iii*) critical angle.
Given $\tan 57° = 1.54$ and $\sin 40.5° = 0.6493$.
[Ans. (*i*) 57° (*ii*) 33° (*iii*) 40.5°]
7. Yellow light is incident on a smooth surface of a block of dense flint glass for which the refractive index is 1.6640. Find the polarising angle and the angle of refraction. (Ans. 59°, 31°)
8. A beam of light travelling in water falls on a glass plate immersed in water. When the incident angle is 51°, the reflected beam of light is found to be completely plane polarised. Determine the refractive index of glass. Given refractive index of water = 4 / 3.
(Ans. 1.647)
9. A ray of light is incident on the surface of a glass plate of refractive index 1.536 at the polarising angle. Calculate the angle of refraction. **[Punjab 95]**
(Ans. 33°4′)
10. The critical angle for a certain wavelength of light in glass is 40°. Calculate the polarising angle and the angle of refraction in glass corresponding to it.
(Ans. 57.3°, 32.7°)

HINTS

3. As $i_p + r_p = 90°$ $\therefore$ $r_p = 90° - i_p = 90° - 53° = \mathbf{37°}$
$\mu = \tan i_p = \tan 53° = \mathbf{1.327.}$
4. As $\tan i_p = \mu = 1.327$ $\therefore$ $i_p = 53°$
$\theta = 90° - i_p = 90 - 53 = \mathbf{37°}.$
7. $\tan i_p = \mu = 1.6640$ $\therefore$ $i_p = \mathbf{59°}.$
$r_p = 90° - 59° = \mathbf{31°}.$
8. ${}^w\mu_g = \tan i_p = \tan 51° = 1.235$
$\therefore {}^a\mu_g = {}^w\mu_g \times {}^a\mu_w = 1.235 \times \frac{4}{3} = \mathbf{1.647.}$
9. $\mu = \tan i_p = 1.536$ $\therefore$ $i_p = \tan^{-1}(1.536) = 56°56'$
$r_p = 90° - 56°56' = \mathbf{33°4'}.$
10. $\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 40°} = 1.5557$
Also $\mu = \tan i_p = 1.5557$ $\therefore$ $i_p = \mathbf{57.3°}$
and $r_p = 90° - 57.3° = \mathbf{32.7°}.$

10.37 POLARISATION BY SCATTERING

43. *Explain polarisation by scattering.*

Polarisation by scattering. If we look at the blue portion of the sky through a polaroid and rotate the polaroid, the transmitted light shows rise and fall of instensity. This shows that the light from the blue portion of the sky is plane polarised. This is because sunlight gets scattered (*i.e.*, its direction is changed) when it encounters the molecules of the earth's atmosphere. The scattered light seen in a direction perpendicular to the direction of incidence is found to be plane polarised.

Explanation. Fig. 10.46 shows the unpolarised light incident on a molecule. The dots show vibrations perpendicular to the plane of paper and double arrows show vibrations in the plane of paper. The electrons in the molecule begin to vibrate in both of these directions. The electrons vibrating parallel to the double arrows cannot send energy towards an observer looking at 90° to the direction of the sun because their acceleration has no transverse component. The light scattered by the molecules in this direction has only dots. It is polarised perpendicular to the plane of paper. This explains the polarisation of light scattered from the sky.

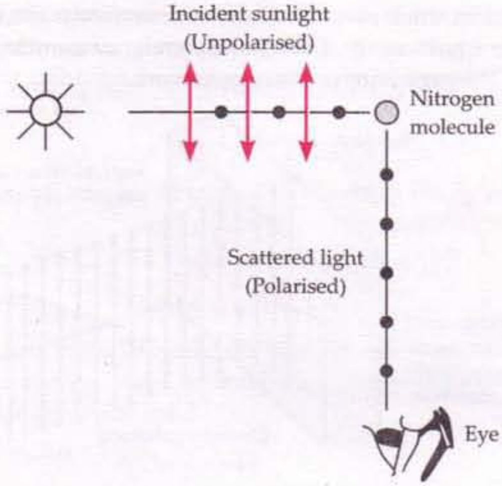


Fig. 10.46 Polarisation by scattering.

For Your Knowledge

- In the nineteen twenties, *C.V. Raman* and his collaborators in Calcutta intensively investigated the scattering of light by molecules. Raman was awarded the Nobel Prize for Physics in 1930 for this work.
- Human eyes cannot distinguish between an unpolarised light and a polarised light. But the eyes of a bee can detect the difference. The bees can, not only, distinguish unpolarised light from polarised light but can also determine the direction of polarisation.

10.38 POLARISATION BY DOUBLE REFRACTION : NICOL PRISM

44. *Explain polarisation by double refraction. Define optic axis and principal section of a doubly refracting crystal.*

Polarisation by double refraction. When an unpolarised ray passes through certain crystals like quartz or calcite, it splits up into two rays, as shown in Fig. 10.47. This phenomenon is called ***double refraction*** or ***birefringence***.

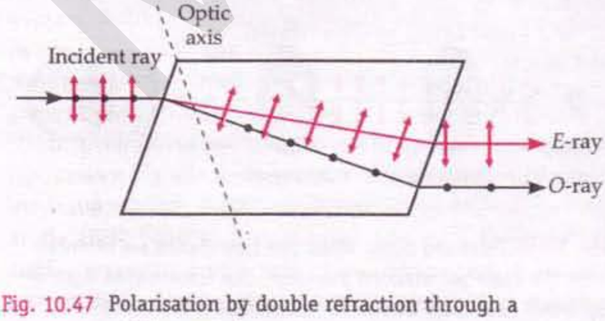


Fig. 10.47 Polarisation by double refraction through a calcite crystal.

1. The one ray which obeys the ordinary laws of refraction and has vibrations perpendicular to the plane of incidence (dots) is called ***O-ray*** or ***ordinary ray***.
2. The other ray which does not obey the laws of refraction and has vibrations parallel to the plane of incidence is called ***E-ray*** or ***extra-ordinary ray***.

Thus both the refracted rays are plane polarised in mutually perpendicular directions. To illustrate the phenomenon of double refraction, make an ink dot on a white paper and look it through a calcite crystal. Two images are seen. If the crystal is rotated about the direction of incident light, it is seen that the image due to *O*-ray remains stationary while the image due to the *E*-ray rotates in the direction of rotation of the crystal.

Optic axis. *It is a particular direction in the crystal along which both the O- and E-rays have equal value of refractive index of the crystal and travel with the same velocity and hence there is no double refraction in this direction.* Along this direction, the images due to *O*-and *E*-rays coincide.

Principal section. *Any plane which contains the optic axis and is perpendicular to the two opposite refracting faces of a crystal is called a principal section of the crystal.*

45. *Explain the principle, construction and working of a nicol prism.*

Nicol Prism. *It is an optical device based on the phenomenon of double refraction which is used for producing and analysing plane polarised light.* It was invented by *William Nicol* in 1828.

Principle. When a thin film of Canada balsam is placed between two calcite pieces, the *O*-rays of the unpolarised incident light get eliminated through the phenomenon of total internal reflection while the *E*-rays are transmitted unaffected and emerge as a beam of plane polarised light.

Construction. The nicol prism consists of two calcite crystals cut at 68° angle with its principal axis joined by a glue called Canada balsam. Canada balsam has a refractive index of 1.55, while the refractive index of calcite for the *O*-rays is 1.658 and that for *E*-rays is 1.486. Thus, Canada balsam acts as a rarer medium for *O*-rays and a denser medium for *E*-rays.

Working. Fig. 10.48 shows the principal section *ACGE* of a nicol prism. The diagonal *AG* represents the Canada balsam layer. When a ray of unpolarised light passes from a portion of the calcite crystal into the layer of Canada balsam, it passes from a denser to a rarer medium. When the angle of incidence is greater than the critical angle (≈ 69°), the ray is totally reflected

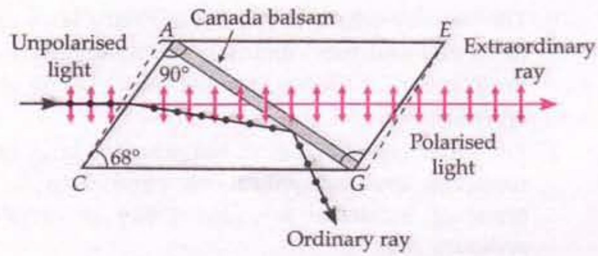


Fig. 10.48 Nicol prism as a polariser.

and absorbed by a blackened surface. The E-ray is not affected because it is travelling from rarer medium (calcite) to denser medium (Canada balsam). It gets transmitted through the nicol prism. Hence a ray of unpolarised light on passing through the nicol prism becomes plane polarised containing vibrations parallel to the principal section.

For Your Knowledge

- **Quarter-wave and half-wave plates.** For a double refracting crystal, the refractive indices for O-ray and E-ray are different and are denoted by μ_0 and μ_e, respectively. When these rays pass through a slab of thickness t, the path difference introduced between the two rays is

$$p = t(\mu_0 \sim \mu_e)$$

A plate which introduces a path difference of $\lambda/4$ between O-rays and E-rays is called a ***quarter wave plate***. The thickness of a quarter wave plate is

$$t_{1/4} = \frac{\lambda}{4(\mu_0 \sim \mu_e)}$$

A plate that introduces a path difference of $\lambda/2$ (or a phase difference of π) between O-rays and E-rays is called a ***half-wave plate***. The thickness of a half-wave plate is

$$t_{1/2} = \frac{\lambda}{2(\mu_0 \sim \mu_e)}$$

10.39 POLARISATION BY SELECTIVE ABSORPTION : DICHROISM

46. *Explain polarisation by selective absorption. Or, explain dichroism.*

Polarisation by selective absorption or dichroism. Certain doubly refracting crystals have the property of absorbing one of the doubly refracted beams to a greater extent than the other. The crystals showing this property are said to be *dichroic* and the phenomenon is known as *dichroism*. *Tourmaline* is a naturally occurring crystal which shows this phenomenon of selective absorption. As shown in Fig. 10.49, when unpolarised light is passed through a tourmaline crystal of sufficient thickness, the O-ray is completely absorbed while the E-ray is almost completely transmitted. So the emergent light is plane polarised.

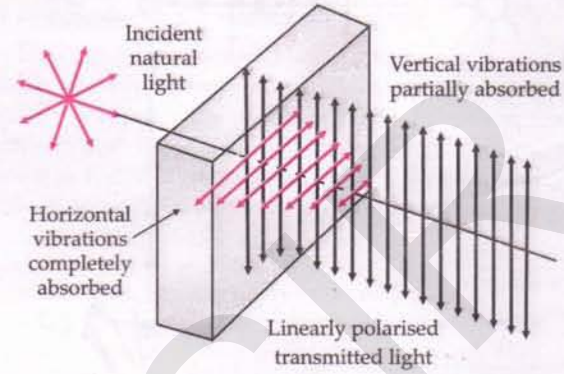


Fig. 10.49 Polarisation by selective absorption.

10.40 POLAROIDS

47. *What are polaroids ? How do they plane polarise an unpolarised beam of light ?*

Polaroids. *Polaroids are thin commercial sheets which make use of the property of selective absorption to produce an intense beam of plane polarised light.*

In 1932, an American scientist *Edwin Land* developed a polariser in the form of large sheets. When a paste of quinine iodosulphate made in nitrocellulose is squeezed out through a fine slit, the needle-shaped crystals of quinine iodosulphate align themselves parallel to their optic axis. These crystals are highly dichroic. They absorb one of the doubly refracted beams completely. The thin polarising sheet so obtained is enclosed between two thin glass plates for mechanical support and we get a polaroid. Each polaroid has a characteristic

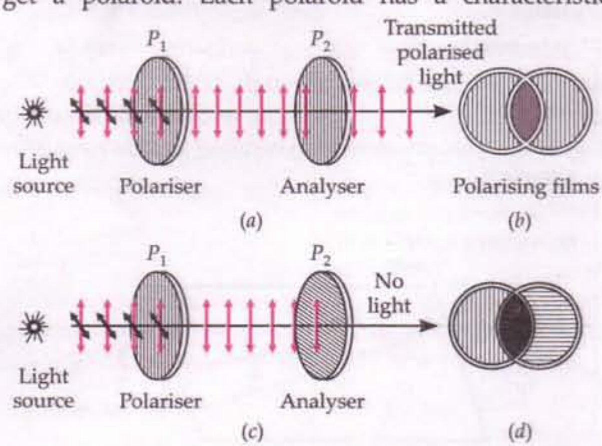


Fig. 10.50 Polarised films. When the film sheets are oriented with the same polarisation direction, the transmitted light is polarised (a and b). When one of the sheets is rotated 90° (crossed polaroids) no light is transmitted (c and d).

direction called polaroid axis (shown by parallel lines). A polaroid transmits only those vibrations which are parallel to its polaroid axis.

As shown in Fig. 10.50, when a beam of unpolarised light falls on a polaroid P_1, it transmits only those vibrations which are parallel to its polaroid axis. It absorbs the vibrations in the perpendicular direction. Thus the transmitted light is plane-polarised. This can be examined by using a second polaroid P_2. When the polaroid axes of the two polaroids are parallel to each other [Figs. 10.50(*a*) and (*b*)], the plane-polarised light transmitted by P_1 is also transmitted by P_2, when the second polaroid is rotated through 90° (***cross polaroids***), no light is transmitted by P_2 [Figs. 10.50(*c*) and (*d*)].

For Your Knowledge

- Improved polaroid films have been developed by using polymer materials. If a film of polyvinyl alcohol (PVA) is stretched to 3 to 8 times its original length, its molecules get oriented in the direction of stress and the film becomes doubly refracting.

  When the stretched film of PVA is impregnated with iodine, it becomes dichroic. The polaroid film so obtained is called ***H-polaroid***.

  If instead of impregnating with iodine, the stretched film is heated in the presence of a strong dehydrating agent, it becomes strongly dichroic and very stable. This polaroid is called ***K-polaroid***.

10.41 USES OF POLAROIDS

48. *Explain some of the important uses of polaroids.*

Uses of polaroids. Polaroids have several uses in daily life :

1. ***In sunglasses and camera filters.*** *Sunglasses* and *camera filters* are made of polarising sheets to reduce the glare of light produced by reflection from shiny surface such as water surface.

2. ***In wind screens.*** The wind screens and car head lights of motor cars are fitted with polaroid films with their axes inclined at 45° to the horizontal. When two cars approach each other from opposite directions, the transmission planes of their wind screens will be perpendicular to each other, so the glare of their head lights is completely eliminated. Each driver sees the road by the light sent by his own car.

3. ***In window panes of aeroplanes.*** One of the polaroids is fixed while the other can be rotated to control the amount of light coming in.

4. ***In photoelasticity.*** Glass and some plastic materials exhibit double refraction only when stressed. If polarised light is passed through them and then analysed, the bright coloured lines indicate the existence of strains. In engineering work, plastic models of structures are constructed and weaknesses are examined in this way.

5. ***In three-D movies.*** Three-D motion pictures are projected on screen by two projectors, each forming a slightly different image. One image is for one eye and other for the second eye so that the brain interprets this difference as depth or third dimension. Lights from each projector are plane polarised, but in mutually perpendicular directions. The two 3-D glasses are really polarising glasses with their directions of polarisation perpendicular to each other. So one eye sees one image and other sees a slightly different image.

6. ***In liquid crystal displays (LCDs).*** An important application of polarisation is in liquid crystal displays or LCDs, used in many watches, calculators and portable computers (lap tops). Liquid crystals have long molecules whose directions can be controlled by applying electric fields. This fact is used in rotating the plane of polarisation of light produced by a polariser so that its polarisation is perpendicular to the axis of an analyser which cuts it out. These dark regions can be controlled with applied voltages and used to form letters and numbers.

10.42 DETECTION OF PLANE POLARISED LIGHT BY POLAROIDS

49. *How can we distinguish between unpolarised, plane-polarised and partially polarised lights by using a polaroid ?*

Detection of unpolarised and polarised lights. The given light is passed through a polaroid and the polaroid is rotated about the direction of incident light. The intensity of the emergent light is observed.

1. If on rotating the polaroid through one complete rotation, there is no change in the intensity of emergent light, then the given light is ***unpolarised***.
2. If the intensity of emergent light shows alternate rise and fall and becomes twice maximum and twice zero in one complete rotation of the polaroid, then the given light is ***plane-polarised*** or ***linearly polarised***.
3. If the intensity of emergent light becomes twice maximum and twice minimum (and not zero) in one complete rotation of the polaroid, then the given light is ***partially polarised***.

10.43 DOPPLER EFFECT OF LIGHT*

50. *State Doppler effect in light. Derive an expression for the apparent frequency of light waves, when source* and observer are in relative motion to each other. What are blue shift and red shift ? Mention the applications of Doppler effect in light.

Doppler effect. In class XI, you have already learnt Doppler effect for sound waves. When a source of sound travels towards an observer, the apparent frequency is higher than the frequency actually emitted by the source. When the source moves away, the apparent frequency is lower than the actual frequency. Doppler effect is a basic property of all waves and so occurs in case of light also.

*Whenever there is a relative motion between source of light and observer, the frequency of light received by the observer is different from the frequency actually emitted by the source. This phenomenon of the apparent change in the frequency of light is called **Doppler effect** for light.*

Expression for the apparent frequency of light. Suppose a source of light emits waves of frequency ν and wavelength λ. If c is the speed of light, then

$$\lambda = \frac{c}{\nu}$$

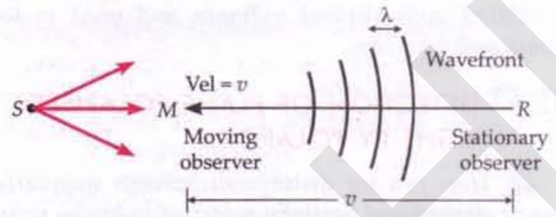


Fig. 10.51

Suppose an observer moves towards the source with velocity v. In one second, the source and observer come closer by a distance v.

$\therefore$ Apparent frequency

= No. of light waves emitted per second by the source + No. of light waves contained in distance v

or $$\nu' = \nu + \frac{v}{\lambda} = \nu + \frac{v}{c/\nu} = \nu + \nu . \frac{v}{c}$$

or $$\nu' = \nu\left(1 + \frac{v}{c}\right) \quad ...(1)$$

Clearly, $\nu' > \nu$ *i.e.*, the apparent frequency increases when source and observer approach each other.

When source and observer move away from each other, the apparent frequency can be obtained by replacing v by $-v$ in the above equation. Then

$$\nu' = \nu\left(1 - \frac{v}{c}\right) \quad ...(2)$$

Clearly, $\nu' < \nu$ *i.e.*, the apparent frequency decreases when source and observer move away from each other.

Blue shift and red shift. Equations (1) and (2) can be combined together as

$$\nu' = \nu\left(1 \pm \frac{v}{c}\right)$$

or $$\nu' - \nu = \pm \frac{v}{c} . \nu$$

The frequency change $\Delta\nu = \nu' - \nu$ is called *Doppler shift*. Putting $\nu = \frac{c}{\lambda}$ and $\nu' = \frac{c}{\lambda'}$, we get

$$\frac{c}{\lambda'} - \frac{c}{\lambda} = \pm \frac{v}{c} . \frac{c}{\lambda}$$

or $$\frac{\lambda - \lambda'}{\lambda'} = \pm \frac{v}{c}$$

But $$\frac{\lambda - \lambda'}{\lambda'} \simeq \frac{\lambda - \lambda'}{\lambda}$$

$\therefore$ $$\frac{\lambda - \lambda'}{\lambda} = \pm \frac{v}{c}$$

or $$\lambda - \lambda' = \pm \frac{v}{c} \lambda$$

(*i*) When source and observer approach each other, positive sign is taken. Then $\lambda - \lambda'$ is positive or $\lambda' < \lambda$, *i.e.*, the wavelengths in the middle part of the visible spectrum shift towards the blue region. This is called **blue shift.**

(*ii*) When source and observer move away from each other, negative sign is taken. Then $\lambda - \lambda'$ is negative or $\lambda' > \lambda$, *i.e.*, the wavelengths in the middle part of the visible spectrum shift towards the red region. This is called **red shift.**

Applications of Doppler effect :

1. Light received from stars and galaxies shows a red shift which indicates that the universe is expanding.
2. By measuring Doppler shift in the e.m. wave reflected from an automobile, the speed of the automobile can be determined.
3. Doppler shift of light received from Saturn rings shows that the rings consist of a number of discontinuous satellites.
4. By measuring Doppler shift in the light received from eastern and western edges of the sun, the speed of rotation of the sun has been determined to be 2 km s^{-1} from east to west relative to the earth.

Examples based on Doppler Effect of Light

Formulae Used

1. $\dfrac{\Delta \nu}{\nu} = \dfrac{\nu' - \nu}{\nu} = \pm \dfrac{v}{c}$
2. $\dfrac{\Delta \lambda}{\lambda} = \dfrac{\lambda' - \lambda}{\lambda} = \mp \dfrac{v}{c}$

Units Used

Frequencies ν and ν' are in hertz, velocities v and c in ms^{-1} and wavelength λ in metre.

Constant Used

Speed of light in free space $= 3 \times 10^8\ ms^{-1}$.

Example 86. *What speed should a galaxy move with respect to us so that the sodium line at 589.0 nm is observed at 589.6 nm?* [NCERT]

Solution. Here, $\lambda = 589.0$ nm,

$$\Delta\lambda = \lambda' - \lambda = 589.6 - 589.0 = 0.6 \text{ nm}$$

As $\Delta\lambda = -\dfrac{v}{c}\lambda$

$$\therefore \quad v = -\frac{\Delta\lambda}{\lambda}.c = -\frac{0.6}{589.0} \times 3 \times 10^8$$

$$= \mathbf{-3.06 \times 10^5\ ms^{-1} = -306\ km/s}$$

The negative sign shows that the galaxy is moving away from us.

Example 87. *The spectral line for a given element in light received from a distant star is shifted towards longer wavelength side by 0.025%. Calculate the velocity of star in the line of light.* [CBSE OD 97]

Solution. Given $\dfrac{\Delta\lambda}{\lambda} = 0.025\% = \dfrac{0.025}{100}$

Velocity of star in the line of sight is

$$v = -\frac{\Delta\lambda}{\lambda} \times c = -\frac{0.025}{100} \times 3 \times 10^8 = \mathbf{-7.5 \times 10^4\ ms^{-1}}$$

The negative shows that the star is receding.

Example 88. *The earth is moving towards a fixed start with a velocity of 30 km s^{-1}. An observer on the earth observes a shift of 0.58 Å in the wavelength of light coming from the star. Find the actual wavelength of light emitted by the star.*

Solution. Here $v = 30\ km\ s^{-1} = 30 \times 10^3 ms^{-1}$, $\Delta\lambda = 0.58$ Å, $c = 3 \times 10^8\ ms^{-1}$

As $\quad \Delta\lambda = \dfrac{v}{c}.\lambda$.

$$\therefore \quad \lambda = \frac{c}{v}.\Delta\lambda = \frac{3 \times 10^8}{30 \times 10^3} \times 0.58 \text{ Å} = \mathbf{5800\ Å}.$$

Example 89. *A radar wave has frequency of 8.1×10^9 Hz. The reflected wave from an aeroplane shows a frequency difference of 2.7×10^3 Hz on the higher side. Deduce the velocity of aeroplane in the line of sight.*

Solution. Here $\nu = 8.1 \times 10^9$ Hz, $\Delta\nu = 2.7 \times 10^3$ Hz

As $\quad \Delta\nu = \dfrac{v}{c}.\nu$

$$\therefore \quad v = \frac{\Delta\nu}{\nu}.c = \frac{2.7 \times 10^3}{8.1 \times 10^9} \times 3 \times 10^8 = 100\ ms^{-1}$$

Since the velocity of the aeroplane determined by radar waves is double of its actual velocity of approach, therefore,

Actual velocity of the aeroplane

$$= 50\ ms^{-1} = \mathbf{180\ km\ h^{-1}}.$$

Problems For Practice

1. Light from a galaxy, having wavelength of 6000 Å, is found to be shifted towards red by 50 Å. Calculate the velocity of recession of the galaxy. [CBSE D 99] (Ans. $2.5 \times 10^6\ ms^{-1}$)
2. The spectral line in the spectrum of light from a star is found to be shifted by 0.032% from its normal position towards the red end of the spectrum. Compute the velocity of the star. (Ans. 96 km s^{-1})
3. A star is moving away from an observer with a speed of 500 kms^{-1}. Calculate the Doppler shift if the wavelength of light emitted by the star is 6000 Å. (Ans. Increase of 10 Å)
4. A star is moving towards the earth with a speed of $9.0 \times 10^6 ms^{-1}$. If the wavelength of a particular spectral line emitted by it is 6000 Å, then find the apparent wavelength. (Ans. 5820 Å)

HINTS

1. $v = -\dfrac{\Delta\lambda}{\lambda}.c = -\dfrac{50}{6000} \times 3 \times 10^8 = \mathbf{-2.5 \times 10^6\ ms^{-1}}$.

2. Here, $\dfrac{\Delta\lambda}{\lambda} = \dfrac{0.032}{100} = 3.2 \times 10^{-4}$

$$v = -\frac{\Delta\lambda}{\lambda}c = -3.2 \times 10^{-4} \times 3 \times 10^8$$

$$= -9.6 \times 10^4 ms^{-1} = \mathbf{-96\ km\ s^{-1}}$$

The negative sign indicates that star is moving away from the earth.

3. $\Delta\lambda = \dfrac{v}{c}.\lambda = \dfrac{500 \times 10^3}{3 \times 10^8} \times 6000 \text{ Å} = 10 \text{ Å}$

4. $\Delta\lambda = \dfrac{v}{c}.\lambda = \dfrac{9.0 \times 10^6}{3 \times 10^8} \times 6000 \text{Å} = 180 \text{ Å}$

Apparent wavelength,

$$\lambda' = \lambda - \Delta\lambda = 6000 - 180 = \mathbf{5820\ Å}.$$

VERY SHORT ANSWER CONCEPTUAL PROBLEMS

Problem 1. What are the reasons to believe that light is a wave motion ? [Punjab 2000]

Solution. Light undergoes interference, diffraction and polarisation. These phenomena establish that light is a wave motion.

Problem 2. How much is the phase difference corresponding to path difference of λ of two waves reaching a point ?

Solution. 2π radian.

Problem 3. *A* and *B* are two points on a water surface where waves are generated. What is the phase difference if (*a*) *A* and *B* are on the same wavefront but separated in distance by the wavelength λ. (*b*) If *A* and *B* are on successive crests, but linearly separated by the distance 2.5 λ and (*c*) *A* and *B* are on successive troughs ?

Solution. (*a*) Zero (*b*) 2π (*c*) 2π.

Problem 4. Is it necessary that the amplitude be constant over a given wavefront ?

Solution. Yes. In a homogeneous medium, the amplitude is constant on a given wavefront.

Problem 5. Can two wavefronts cross one another ? Give reason.

Solution. No. If they intersect, then there will be two rays or two directions of propagation of energy at the point of intersection which is not possible.

Problem 6. A light wave enters from air into glass. How will the following be affected :

(*i*) Energy of the wave

(*ii*) Frequency of the wave ? [CBSE F 94]

Solution. (*i*) Energy of the wave decreases because a part of the light wave is reflected back into air.

(*ii*) Frequency of the wave remains unchanged.

Problem 7. When a wave undergoes reflection at a denser medium, what happens to its phase ?

Solution. When a wave is reflected into rarer medium from the surface of a denser medium, it undergoes a phase change of π radian.

Problem 8. If a wave undergoes refraction, what will be the phase change ?

Solution. Zero. No phase change occurs during refraction.

Problem 9. When monochromatic light travels from one medium to another its wavelength changes but frequency remains the same. Explain. [CBSE D 11]

Solution. Frequency is the characteristic of the source while wavelength is the characteristic of the medium. When monochromatic light travels from one medium to another, its speed changes, so its wavelength ($\lambda = c/\nu$) changes but frequency ν remains unchanged.

Problem 10. Define the term 'coherence' for light waves.

Solution. Two light waves interfering with each other are said to show coherence if the initial phase difference between them remains constant with time.

Problem 11. State the essential conditions for two light waves to be coherent. [Haryana 02, 04 ; CBSE OD 94C]

Solution. 1. The two waves must be continuous.

2. The two waves should be of same frequency or wavelength.
3. They should have a constant or zero phase difference.
4. Preferably, they should have equal amplitude.

Problem 12. State two conditions to obtain sustained interference of light. [CBSE D 02, 04, F 08]

Solution. The conditions for obtaining sustained interference of light are (*i*) The two light sources should be coherent. (*ii*) The two light sources should be narrow and placed close to each other.

Problem 13. What happens to the interference pattern if the phase difference between the two sources varies continuously ?

Solution. The positions of bright and dark fringes will change rapidly. Such rapid changes cannot be detected by our eyes. A uniform illumination is seen on the screen *i.e.*, interference pattern disappears.

Problem 14. Why are coherent sources necessary to produce a sustained interference pattern ? [CBSE F 09 ; D 12 ; OD 13C]

Solution. Coherent sources have a constant phase difference. This ensures that the positions of maxima and minima do not change with time *i.e.*, a sustained interference pattern is obtained.

Problem 15. Two slits in Young's double slit experiment are illuminated by two different lamps emitting light of the same wavelength. Will you observe the interference pattern ? Justify your answer. [CBSE OD 03]

Solution. No. The light waves emitted by two different lamps cannot be coherent. So the conditions of maxima and minima of intensity will change rapidly on the screen, producing uniform illumination.

Problem 16. 'Two independent monochromatic sources of light cannot produce a sustained interference pattern'. Give reason. [CBSE D 14 ; F 15]

Solution. The phase difference between the light waves originating from two independent monochromatic sources will change rapidly with time. The two sources will not be coherent and, therefore, will not produce a sustained interference pattern.

Problem 17. **A slit, S is illuminated by a monochromatic source of light to give two coherent sources P_1 and P_2. These give bright and dark bands on a screen. At a point R, on the screen, there is a dark fringe. What relationship must exist between the lengths P_1R and P_2R ?** [CBSE D 93C]

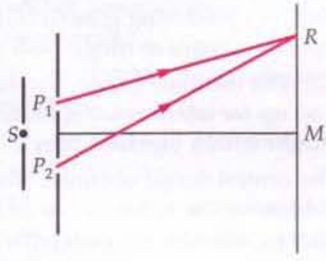


Fig. 10.52

Solution. The condition for the dark fringe is

$$p = P_2 R - P_1 R = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3 \ldots$$

Problem 18. **If the separation between the two slits is decreased in Young's double slit experiment keeping the screen position fixed, what will happen to the fringes width ?** [ISCE 01]

Solution. Fringe width, $\beta = \frac{D\lambda}{d}$

As the separation 'd' between the two slits decreases, fringe width β increases.

Problem 19. **How does the fringe width, in Young's double-slit experiment, change when the distance of separation between the slits and screen is doubled ?** [CBSE OD 12]

Solution. Fringe width, $\beta = \frac{D\lambda}{d}$

When the distance D between the slits and the screen is doubled, the fringe width also gets doubled.

Problem 20. **Why should we have a narrow source to produce good interference fringes ?** [Haryana 1993]

Solution. It is because a broad source is equivalent to a large number of narrow sources lying close to each other. Different pairs of narrow sources will produce their own interference patterns which will overlap each other. So the fringe system is lost.

Problem 21. **In Young's double slit experiment if the distance between two slits is halved and distance between the slits and the screen is doubled, then what will be the effect on fringe width ?**

Solution. Original fringe width, $\beta = \frac{\lambda D}{d}$

New fringe width, $\beta' = \frac{\lambda \cdot 2D}{d/2} = \frac{4 \times D}{d} = 4\beta$

i.e., fringe width increases four times.

Problem 22. **No interference pattern is detected when two coherent sources are infinitely close to one another. Why ?**

Solution. Fringe width, $\beta = \frac{\lambda D}{d}$

i.e., $\beta \propto \frac{1}{d}$, when $d \to 0$, $\beta \to \infty$

Fringe width is very large. Even a single fringe may occupy the entire screen. The interference pattern cannot be observed.

Problem 23. **Why is interference pattern not detected, when the two coherent sources are far apart ?** [Haryana 01 ; CBSE OD 03]

Solution. When the distance d between the two coherent sources is large, the fringe width $(\beta \propto 1/d)$ becomes too small to be detected. The interference pattern cannot be observed.

Problem 24. **If the path difference produced due to interference of light coming out of two slits for yellow colour of light at a point on the screen be $3\lambda/2$, what will be the colour of the fringe at the point ? Give reason also.**

Solution. The given path difference satisfies the condition for the minimum of intensity for yellow light. Hence if yellow light is used, a dark fringe will be formed at the given point. If white light is used, all components of white light except the yellow one would be present at this point.

Problem 25. **In Young's double slit experiment, lights of green, yellow and orange colours are successively used. Write the fringe widths for the three colours in increasing order.**

Solution. Fringe width, $\beta = \frac{\lambda D}{d}$ *i.e.*, $\beta \propto \lambda$

As $\lambda_G < \lambda_Y < \lambda_0$, therefore, $\beta_G < \beta_Y < \beta_0$.

Problem 26. **When a thin transparent film is placed just in front of one of the slits in the Young's double slit experiment using white light, what change results in the fringe system ?** [ISCE 97]

Solution. If μ is the refractive index and t the thickness of the thin film, then the entire interference pattern gets displaced by distance,

$$\Delta x = \frac{D}{d}(\mu - 1)t$$

As μ depends on λ (increases with decreasing λ), the violet fringe is shifted farther than the red fringe. So there is a kind of dispersion in the central white fringe.

Problem 27. **How will the intensity of maxima and minima, in the Young's double slit experiment change, if one of the two slits is covered by a transparent paper which transmits only half of the light intensity ?** [CBSE Sample Paper 11]

Solution. Intensity of maxima decreases and that of minima increases.

Problem 28. State with reason whether diffraction of light takes place at the two slits in the Young's interference experiment.

Solution. Yes, the light waves suffer diffraction at both the slits. Then interference occurs between the diffraction patterns of the two slits.

Problem 29. Explain the statement 'light added to light can produce darkness'. [CBSE D 92C]

Solution. When two light waves of equal amplitude meet at a point in opposite phases, the resultant amplitude and hence intensity become zero at that point. That is when light added to light undergoes destructive interference, it produces darkness.

Problem 30. What happens to the light energy when light waves interfere destructively at a point ?

Solution. Energy gets transferred from the regions of destructive interference to the regions of constructive interference.

Problem 31. What will be the effect on the fringes formed in Young's double slit experiment, if the apparatus is immersed in water ? [CBSE D 99]

Solution. The wavelength of light in water ($\lambda' = \lambda / \mu$) is less than that in air. When the apparatus is immersed in water, fringe width ($\beta \propto \lambda'$) decreases.

Problem 32. Why is it comparatively difficult to observe interference in light waves as compared to that in water waves ?

Solution. This is because the wavelength of light waves is much smaller than the wavelength of water waves. Consequently, the interference fringes have much smaller width in case of light waves than in water waves.

Problem 33. Is there any difference between the colours emerging from a prism and the colours of a soap film seen in sunlight ?

Solution. Yes. In the prism, colours are produced due to dispersion of light. The colours of a soap film are due to interference of light.

Problem 34. Why does a soap bubble show beautiful colours when illuminated by white light ? [CBSE D 94C ; Punjab 04]

Solution. Light waves reflected from the upper and lower surfaces of a thin film interfere. Since the conditions for bright and dark fringes are satisfied at different positions for different wavelengths, so coloured fringes are observed.

Problem 35. Why does an excessively thin film appear black in reflected light ? [Haryana 92]

Solution. For an excessively thin film ($t << \lambda$), the factor $2\,\mu t \cos r$ is negligibly small. The effective path difference between any two successive rays in reflected system is $\lambda/2$. This is the condition for minimum intensity and hence the film will appear dark. But the film appears bright in transmitted light.

Problem 36. Why do thick films not show interference effects ?

Solution. When the film is thick ($t \simeq 20\,\lambda$), the path difference $2\,\mu t \cos r$ will be so large that the conditions of both maxima and minima for different wavelengths of white light will be essentially satisfied at the same value of thickness t. Different colours will overlap each other at all places of the film ; producing general illumination of the film. No separate colours or fringes will be seen.

Problem 37. The central fringe obtained with a Lloyd's mirror set up for interference is found to be dark, whereas it is bright with a biprism. Why ? [ISCE 03]

Solution. The central fringe obtained with a Lloyd's mirror is dark because the reflection of light from the denser mirror surface introduces a path difference of $\lambda / 2$. In case of biprism, the waves from both the coherent sources reach the central point in the same phase and so the central fringe is bright.

Problem 38. State the essential condition for diffraction of light to occur. [CBSE OD 01]

Solution. Diffraction of light occurs when size of the obstacle or the aperture is comparable to the wavelength of light.

Problem 39. What should be the order of size of obstacle or aperture for diffraction of light ? [Haryana 95]

Solution. The size of the obstacle or aperture should be of the order of the wavelength of light used.

Problem 40. How does the angular separation between fringes in single-slit diffraction experiment change when the distance of separation between the slit and screen is doubled ? [CBSE OD 12]

Solution. Angular separation between diffraction fringes,

$$\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

When the distance (D) between the slit and screen is doubled, the angular separation θ remains unchanged.

Problem 41. A parallel beam of monochromatic light falls normally on a narrow slit of width '*a*' to produce a diffraction pattern on the screen placed parallel to the plane of the slit. Use Huygens' principle to explain that the central bright maxima is twice as wide as the other maxima. [CBSE D 14C]

Solution. In diffraction pattern, intensity will be minimum at an angle $\theta = n\lambda / a$. So there will be a first minimum at an angle $\theta = \lambda / a$, on either side of central maximum.

Hence, width of central maximum = $2\lambda / a$, whereas the width of secondary maxima/minima = λ / a.

Problem 42. Why does the intensity of the secondary maximum become less as compared to the central maximum ? [CBSE OD 09, D 14C]

Solution. The central maximum is due to the constructive interference of wavelets from all parts of the slit. With the increase in the value of n, the wavelets from lesser and lesser parts of the slit produce constructive interference to form a secondary maximum. Hence the intensity of secondary maximum decreases with the increase in the value of n.

Problem 43. For a single slit of width "a", the first minimum of the interference pattern of a monochromatic light of wavelength λ occurs at an angle of $\frac{\lambda}{a}$. At the same angle of $\frac{\lambda}{a}$, we get a maximum for two narrow slits separated by a distance "a". Explain. [CBSE D 14]

Solution. In the first case, the corresponding wavelets from the two halves of the slit have a path difference of $\frac{\lambda}{2}$, so their overlapping produces first minimum.

In the second case, the corresponding wavelets from the two slits have the path difference of λ, so their overlapping produces first maximum.

Problem 44. Yellow light is used in a single slit diffraction experiment with slit width of 0.6 mm. If yellow light is replaced by X-rays, how will the diffraction pattern be affected ? [IIT 99]

Solution. As the wavelength of X-rays is much smaller than that of yellow light, so the diffraction pattern is lost when the yellow light is replaced by X-rays.

Problem 45. Is it correct to say that diffraction is interference between different parts of the same wavefront ?

Solution. Yes, diffraction is due to the interference of secondary wavelets starting from different parts of the wavefront that passes through an aperture or that is unobstructed by the obstacle.

Problem 46. A small circular disc is placed in the path of light from a distant source. Will the centre of shadow be bright or dark ?

Solution. Waves from distant source are diffracted by the edge of the disc. These diffracted waves interfere constructively at the centre of the shadow and produce a bright fringe.

Problem 47. Why do we fail to observe the diffraction from a wide slit illuminated by monochromatic light ?

Solution. When the size of the slit is large, the central maximum is small in size and the variation in the intensity of the other maxima and minima is so small that they cannot be distinguished. That is why we fail to observe the diffraction pattern. Instead, we see a sharp image of the object.

Problem 48. Diffraction is common in sound but not common in light waves ? Why ? [Haryana 94 ; CBSE D 97, 02]

Or

Why do we not encounter diffraction effects of light in everyday observations ? [CBSE F 09]

Solution. Diffraction effect is more pronounced if the size of obstacle or aperture is of the order of the wavelength of the waves. As the wavelength of light ($\sim 10^{-6}$ m) is much smaller than the size of the objects around us, so diffraction of light is not easily seen. But sound waves have large wavelength. They get easily diffracted by the objects around us.

Problem 49. Radiowaves diffract pronouncedly around the buildings, while light waves, which are e.m. waves, do not. Why ?

Solution. For diffraction to take place, the wavelength should be of the order of the size of the obstacle. The radio waves (particularly short radio waves) have wavelengths of the order of the size of the building and other obstacles coming in their way and hence they easily get diffracted. Since wavelength of the light waves is very small, they are not diffracted by buildings.

Problem 50. If light bends around obstacles, then why can't we see around a building ?

Solution. The wavelength of light is much smaller than the size of the wall. So diffraction of light around the edge of the wall is very poor. We cannot see around the house.

Problem 51. You are able to hear a person standing behind a wall but not see him, though both light and sound are waves. Why ?

Solution. This is because sound waves get easily diffracted round the edge of the wall while light waves do not.

Problem 52. A single slit diffraction pattern is completely immersed in water without changing any other parameter. How is the width of central maximum affected ?

Solution. Wavelength of light in water ($\lambda' = \lambda / \mu$) decreases, so width of central maximum ($\beta_0 \propto \lambda$) also decreases.

Problem 53. Coloured spectrum is seen, when we look through a muslin cloth. Why ? [Himachal 97]

Solution. Muslin cloth is made of very fine threads and as such fine slits are formed. White light passing through these slits gets diffracted giving rise to coloured spectrum. The central maximum is white while the secondary maxima are coloured. This is because the positions of secondary maxima (except central maximum) depend on the wavelength of light.

In a coarse cloth, the slits formed between the threads are wider and the diffraction is not so pronounced. Hence no such spectrum is seen.

Problem 54. Why are diffraction effects more prominent through a slit formed by two blades than through a slit formed by two fingers ?

Solution. Diffraction is prominent when we use a narrow slit having parallel edges. Such a slit can be obtained by using two blades and not by using two fingers.

Problem 55. How does the resolving power of a telescope change when the aperture of the objective is increased ? [CBSE Sample Paper 97]

Solution. R.P. of a telescope $= \dfrac{D}{1.22\,\lambda}$

When the aperture (D) of the objective is increased, the resolving power of the telescope increases.

Problem 56. What is the relation between magnifying power and resolving power of a telescope ?

Solution. Magnifying power

$$= \frac{\text{Resolving power of the eye}}{\text{Resolving power of the telescope}}$$

Problem 57. Why is the resolving power of a microscope having oil immersion objective high ?

Solution. R.P. of a microscope $= \dfrac{2\,\mu\,\sin\theta}{\lambda}$

Since such a microscope uses oil of high refractive index (μ) between the object and the objective, so it has a high resolving power.

Problem 58. Which type of waves show the property of polarisation ? [ISCE 97 ; CBSE OD 2000C]

Solution. Only transverse waves show the property of polarisation.

Problem 59. Which special characteristic of light is demonstrated only by the phenomenon of polarisation ? [CBSE D 04C]

Solution. The phenomenon of polarisation demonstrates that light has transverse wave nature.

Problem 60. Will ultrasonic waves show any polarisation ? Give reasons for your answer. [ISCE 95]

Solution. No, ultrasonic waves are longitudinal in nature, so they cannot be polarised.

Problem 61. Why longitudinal waves cannot be polarised ?

Solution. In polarisation, vibrations perpendicular to the direction of propagation are restricted to just one direction. This is possible in transverse waves which have such vibrations. In longitudinal waves, vibrations occur along the direction of propagation. So their polarisation is not possible.

Problem 62. Light waves can be polarised while sound waves cannot. Why ?

Solution. Only transverse waves can be polarised. Light waves are transverse in nature, so they can be polarised. But sound waves have longitudinal nature, so they cannot be polarised.

Problem 63. Which plane is defined as the plane of polarization in a plane polarized electromagnetic wave ?

Solution. The plane containing the direction of propagation of light and perpendicular to the plane of vibration is called plane of polarisation. It contains no vibrations.

Problem 64. Name three properties, which are mutually perpendicular to each other in a plane, polarized light wave. [ISCE 02]

Solution. Electric field vector, magnetic field vector and direction of propagation of the light wave.

Or

Plane of vibration, plane of polarisation and direction of propagation of the light wave.

Problem 65. Which field vector, electric or magnetic, is used to represent the polarisation of an e.m. wave ?

Solution. Electric field vector.

Problem 66. Why does the electric field of e.m. wave determine the state of polarisation and not its magnetic field ?

Solution. An e.m. wave exerts a much larger electric force on a slowly moving charged particle than the magnetic force. Optical phenomena can be explained by considering the interaction between the electric field vector of light and the matter through which it passes. So one can specify the state of polarisation by electric field vector only.

Problem 67. Does the value of polarising angle of incidence depend on the colour of light ?

Solution. The refractive index of a material depends on the colour or wavelength of light. As the polarising angle depends on refractive index ($\mu = \tan i_p$), so it also depends on wavelength of light.

Problem 68. If the polarizing angle for air-glass interface is 56.3°, what is the angle of refraction in glass ?

Solution. As $i_p + r_p = 90°$

$$\therefore \quad r_p = 90 - i_p = 90° - 56.3° = 33.7°.$$

Problem 69. At what angle of incidence should a light beam strike a glass slab of refractive index $\sqrt{3}$, such that the reflected and the refracted rays are perpendicular to each other ? [CBSE D 97C, 09]

Or

What is the polarising angle of a medium of refractive index $\sqrt{3}$ (or 1.732) ? [CBSE OD 99, 2000]

Solution. When the reflected and refracted rays are perpendicular or at the polaring angle of incidence,

$$\tan i_p = \mu = \sqrt{3}$$

$$\therefore \quad i_p = \tan^{-1}(\sqrt{3}) = 60°.$$

Problem 70. Good quality sun-glasses made of polaroids are preferred over ordinary coloured glasses. Justify your answer. [CBSE OD 15C]

Solution. Polaroid sunglasses are preferred over coloured sunglasses because :

(*i*) they are more effective than coloured glasses in cutting off the harmful UV rays from the sun.

(*ii*) they are more effective in reducing the glare due to reflections from horizontal surfaces.

Problem 71. **Write two simple uses of polaroids.** [Haryana 97 ; Punjab 99C]

Solution. (*i*) Polaroids are used in sunglasses and camera filters to reduce glare of light.

(*ii*) In window panes of aeroplanes to control the amount of light coming in.

Problem 72. **A partially plane polarised beam of light is passed through a polaroid. Show graphically the variation of the transmitted light intensity with angle of rotation of the polaroid.** [CBSE Sample Paper 08]

Solution. On rotating the polaroid, the unpolarised component remains unchanged while the polarised component shows alternate maxima and minima in accordance with the relation, $I = I_0 \cos^2 \theta$.

The net variation of the transmitted light is as shown in Fig. 10.53.

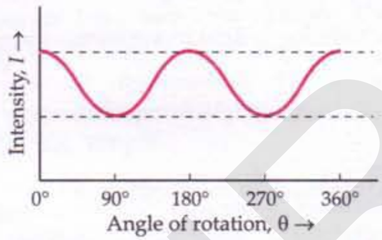


Fig. 10.53

SHORT ANSWER CONCEPTUAL PROBLEMS

Problem 1. **Answer the following questions :**

(*a*) When monochromatic light is incident on a surface separating two media, the reflected and refracted light both have the same frequency as the incident frequency. Explain why. [CBSE D 11, 13]

(*b*) When light travels from a rarer to a denser medium, it loses some speed. Does the reduction in speed imply a reduction in the energy carried by the light wave ? [CBSE OD 10 ; D 13]

(*c*) A narrow pulse of light is sent through a medium. Will you expect the pulse to retain its shape as it travels through the medium ?

(*d*) In the wave picture of light, intensity of light is determined by the square of the amplitude of the wave. What determines the intensity of light in the photon picture of light ?

(*e*) The speed of light in still water is c/μ, where μ is the refractive index of the water. What is the speed of light in a stream of water flowing at a steady speed of v relative to the observer ? [NCERT]

Solution. (*a*) Both reflection and refraction occur due to interaction of light with the atoms at the surface of separation. These atoms may be regarded as oscillators. Light incident on such atoms forces them to vibrate with the frequency of light. As the light emitted by these charged oscillators is equal to their own frequency of oscillation, so both the reflected and refracted lights have the same frequency as the frequency of incident light.

(*b*) No, the reduction in the speed of light does not imply the reduction in the energy of the light wave because the energy carried by a wave depends on the amplitude of the wave, not on the speed of wave propagation.

(*c*) No. A pulse can be viewed as being made of harmonic waves with a large range of wavelengths. Since the speed of propagation in a medium depends on wavelength, different wavelength components of the pulse travel with different speeds. Hence according to Huygen's theory of wave propagation, the pulse will not retain its shape as it travels through the medium.

(*d*) In the photon picture of light, intensity of light at a point is determined by the number of photons incident per unit area around that point.

(*e*) The speed of light in water depends on the relative motion between the observer and water. One may expect the answer to be $\frac{c}{\mu} + v$. According to the special theory of relativity, the correct speed of light in the stream of water is

$$c' = \frac{c}{\mu} + v\left(1 - \frac{1}{\mu^2}\right), \text{ for } v << c.$$

Problem 2. **(*a*) Define a wavefront. How is it different from a ray ?** [CBSE OD 15C]

(*b*) Depict the shape of a wavefront in each of the following cases :

(*i*) **Light diverging from point source.**

(*ii*) **Light emerging out of a convex lens when a point source is placed at its focus.**

(*iii*) **Using Huygens' construction of secondary wavelets, draw a diagram showing the passage of a plane wavefront from a denser into a rarer medium.**

Solution. (*a*) A wavefront is the locus of all such points of the medium which have the same phase.

Differences of a wavefront from a ray : (*i*) A ray is normal to the wavefront at each point. (*ii*) A ray indicates the direction of propagation of a wave while the wavefront is the surface of constant phase.

(*b*) (*i*) See Fig. 10.54(*a*) (*ii*) See Fig. 10.54(*b*)

(*iii*) See Fig. 10.55(*i*).

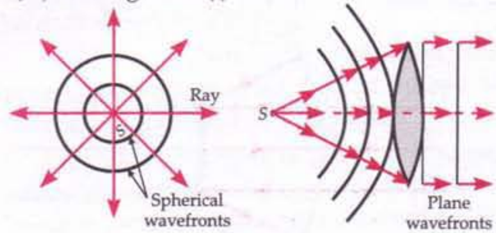


Fig. 10.54 (*a*) (*b*)

Problem 3. If ε_0 and μ_0 are the permittivity and permeability of free space and ε and μ are corresponding quantities for a medium, then show that the refractive index of the medium is $\sqrt{\mu\varepsilon/\mu\varepsilon_0}$.

Solution. Velocity of light in vacuum $= c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Velocity of light in medium $= v = \dfrac{1}{\sqrt{\mu\varepsilon}}$

$\therefore$ Refractive index of the medium, $n = \dfrac{c}{v} = \sqrt{\dfrac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

Problem 4. (a) A plane wavefront approaches a plane surface separating two media. If medium one is (optically) denser and medium two is (optically) rarer, construct the refracted wave front using Huygens' principle. Hence prove Snell's law. [CBSE OD 15]

(b) Draw the shape of the refracted/reflected wavefront when a plane wavefront is incident on (i) prism and (ii) convex mirror. Give a brief explanation for the construction. [CBSE Sample Paper 11]

Solution. (a) The refracted wavefront *CE* is shown in Fig. 10.55(*i*).

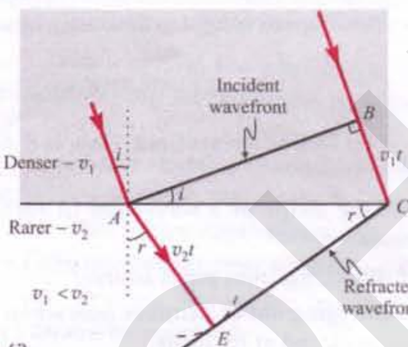


Fig. 10.55 (*i*)

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}\ ; \quad \sin r = \frac{AE}{AC} = \frac{v_2 t}{AC}$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{v_1}{v_2} \quad \text{or} \quad {}^1\mu_2 = \frac{v_1}{v_2} = \text{a constant}$$

This proves Snell's law of refraction.

(*b*)

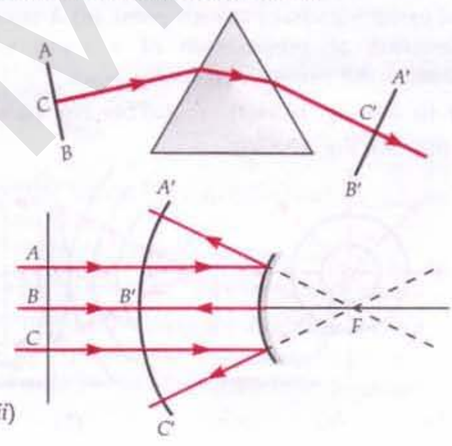


Fig. 10.55 (*ii*)

Explanation. The time taken by any disturbance to travel from incident wavefront to the refracted or reflected wavefront is same.

Problem 5. Two monochromatic waves emanating from two coherent sources have the displacements represented by $y_1 = a\cos\omega t$, and $y_2 = a\cos(\omega t + \phi)$, where ϕ is the phase difference between the two displacements. Show that the resultant intensity at a point due to their superposition is given by $I = 4I_0\cos^2\phi/2$, where $I_0 \propto a^2$. Hence obtain the conditions for constructive and destructive interference. [CBSE D 14 ; OD 14C, 15]

Solution. By the principle of superposition, the resultant displacement at the observation point will be

$$y = y_1 + y_2 = a[\cos\omega t + \cos(\omega t + \phi)]$$
$$= 2a\cos\frac{\phi}{2}.\cos\left(\omega t + \frac{\phi}{2}\right)$$

Amplitude of the resultant displacement $= 2a\cos\dfrac{\phi}{2}$

$\because$ Intensity $\propto$ (amplitude)^2

$\therefore$ Intensity, $\quad I = 4ka^2\cos\dfrac{\phi}{2},$

where k = a proportionality constant.

If I_0 is the intensity of each source, then $I_0 = ka^2$ and

$$I = 4I_0\cos^2\frac{\phi}{2}$$

For constructive interference :

$$\cos\frac{\phi}{2} = \pm 1 \quad \text{or} \quad \frac{\phi}{2} = n\pi \quad \text{or} \quad \phi = 2n\pi$$

For destructive interference :

$$\cos\frac{\phi}{2} = 0 \quad \text{or} \quad \frac{\phi}{2} = (2n+1)\frac{\pi}{2} \quad \text{or} \quad \phi = (2n+1)\pi$$

Problem 6. A region is illuminated by two sources of light. The intensity I at each point is found to be equal to $I_1 + I_2$, where I_1 is the intensity of light at the point when source 2 is absent. I_2 is similarly defined. Are the sources coherent or incoherent ? Explain. [NCERT]

Solution. When two waves of intensities I_1 and I_2 and having phase difference ϕ meet at a point, the resultant intensity is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Given intensity, $I = I_1 + I_2$

$\therefore$ Interference term, $\quad 2\sqrt{I_1 I_2}\cos\phi = 0$

or $\qquad \cos\phi = 0 \qquad$ [I_1 and I_2 are not zero]

That is, the phase difference ϕ varies from 0 to 2π in such a way that the average value of $\cos\phi$ is zero over a cycle. Thus two sources have a phase difference which is not stable. Such sources are called ***incoherent sources***.

Problem 7. What is the effect on the interference fringes in a Young's double-slit experiment due to each of the following operations :

(a) The screen is moved away from the plane of the slits.

(b) The (monochromatic) source is replaced by another (monochromatic) source of shorter wavelength.

(c) The separation between the two slits is increased.

(d) The source slit is moved closer to the double-slit plane.

(e) The width of the source slit is increased.

(f) The widths of two slits are increased.

(g) The monochromatic source is replaced by source of white light ?

(In each operation, take all parameters, other than the one specified, to remain unchanged.) [NCERT]

Solution. In Young's double slit experiment, the fringe width is given by

$$\beta = \frac{D\lambda}{d}$$

(a) When the screen is moved away from the plane of the slits, the distance D increases. Hence fringe width β increases ($\beta \propto D$). But the angular separation ($= \lambda/d$) of the fringes remains constant.

(b) The decrease in wavelength λ decreases the fringe width β ($\beta \propto \lambda$). Also the angular separation of fringes decreases.

(c) As the separation d between the two slits increases, the fringe width β decreases ($\beta \propto 1/d$).

(d) Let s be the width of the source slit and S its distance from the plane of the two slits. For interference fringes to be distinctly seen, the condition

$$\frac{s}{S} < \frac{\lambda}{d}$$

should be satisfied, otherwise, the interference patterns produced by different parts of the source slit will overlap. The minima will not be totally dark and no fringe will be seen. So as the source slit is brought closer, the value of S decreases and the interference pattern becomes less and less sharp. When the source is brought very close so that the above condition gets violated, the fringes disappear. However, as long as the fringes are visible, the fringe width remains constant.

(e) A broad source is equivalent to a large number of narrow sources placed close together. All such narrow sources produce their own interference patterns which overlap with each other and so the fringe pattern becomes less and less sharp. When the source slit is so wide that the condition $\frac{s}{S} < \frac{\lambda}{d}$ is not satisfied, the interference pattern disappears.

(f) The angular size of the central diffraction band due to each slit is about $\frac{\lambda}{S'}$, where S' is the width of each of the two slits. S' should be sufficiently small so that these bands are wide enough to overlap and thus produce interference. This means $\frac{\lambda}{S'} > \frac{\lambda}{d}$, *i.e.*, the width of each slit should be considerably smaller than the separation between the slits. When the slits are so wide that this condition is not satisfied, fringes are not seen. However, increase in the width of the slits does improve the brightness of the fringes. Thus, in practice, the two slits should be wide enough to allow sufficient light to pass through but narrow enough to cause enough diffraction from each slit to enable wavefronts from the two slits to overlap and interfere.

(g) White light consists of colours from violet to red with wavelength from 4000 Å to 7000 Å. The interference patterns due to different component colours of white light overlap. At the centre of the screen, the path difference is zero for all such components. Therefore, the central fringe is white. Since the violet colour has the lowest λ, the closest fringe on either side of the central white fringe is violet, while the farthest fringe is red. After a few fringes, the fringe pattern is lost due to large overlapping.

Problem 8. What will be the effect on the interference fringes in Young's double slit experiment when,

(i) the width of the source slit is increased ?

(ii) the monochromatic sources is replaced by another monochromatic source of shorter wavelength ?

(iii) monochromatic source is replaced by a source of white light ? [CBSE F 08 ; OD 09, 15]

Solution. (i) The interference pattern will become less and less sharp.

(ii) Fringe width, $\beta = \frac{D\lambda}{d}$

With monochromatic source of shorter wavelength λ, fringe width β decreases.

(iii) The central fringe is white. The closest fringe on either side of white fringe is violet (smallest λ), while the farthest fringe is red (largest λ).

Problem 9. What is the effect on the interference pattern observed in a Young's double slit experiment in the following cases : [CBSE OD 2000]

(i) Screen is moved away from the plane of the slits.

(ii) Separation between the slits is increased.

(iii) Widths of the slits are doubled.

Solution. Fringe width, $\beta = \frac{D\lambda}{d}$

(i) As $\beta \propto D$, so when screen is moved away from the slits, fringe width increases.

(ii) As $\beta \propto 1/d$, so when the separation between the slits is increased, fringe width decreases.

(iii) When widths of slits are doubled, contrast between maxima and minima decreases due to the overlapping of interference patterns formed by various narrow pairs of the two slits.

Problem 10. What is the effect on the interference fringes in Young's double slit experiment if (i) the separation between the slits is halved, and (ii) the source slit is moved closer to the double slit ? Justify your answer. [CBSE D 02C]

Solution. (i) Fringe width, $\beta = \frac{D\lambda}{d}$. When separation d between the slits is halved, fringe width is doubled.

(ii) The fringes became indistinct because the interference patterns due to various parts of the source slit overlap.

Problem 11. Why is no interference pattern observed when two coherent sources are

(i) infinitely close to each other ?

(ii) far apart from each other ? [CBSE OD 98]

Solution. Fringe width, $\beta = \frac{D\lambda}{d}$ i.e., $\beta \propto \frac{1}{d}$

(i) When the two coherent sources are placed infinitely close to each other, the fringe width becomes very large. Even a single fringe may occupy the entire screen. The interference pattern is not observable.

(ii) As the distance between the sources is increased, the fringe width goes on decreasing. At very large separation, it becomes too small to be detected. The interference pattern cannot be observed.

Problem 12. What changes in the interference pattern in Young's double slit experiment will be observed when

(i) light of smaller frequency is used ? [CBSE F 94]

(ii) the apparatus is immersed in water ? [CBSE OD 99]

Solution. In Young's double slit experiment, the fringe width is given by

$$\beta = \frac{D\lambda}{d} = \frac{Dc}{d\nu}$$

Clearly,

(i) When light of smaller frequency (ν) is used, fringe width increases.

(ii) Wavelength of light in water decreases, so fringe width ($\beta \propto \lambda$) also decreases.

Problem 13. In a Young's double slit experiment, the position of the first fringe coincides with S_1 and S_2 respectively. What is the wavelength of light ?

Solution. As shown in Fig. 10.56, the bright fringes B_1 and B_2 on either side of O coincide with S_1 and S_2 respectively.

Clearly,

$$\beta = \frac{d}{2}$$

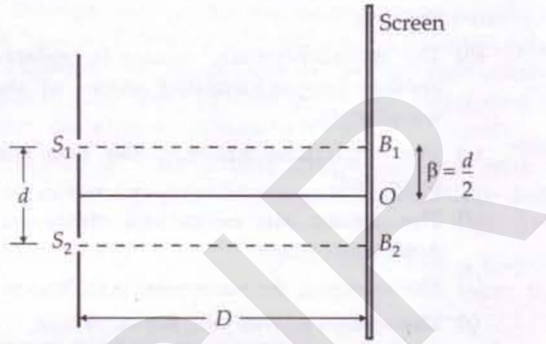


Fig. 10.56

As $\beta = \frac{D\lambda}{d}$ $\therefore$ $\frac{d}{2} = \frac{D\lambda}{d}$

or $\lambda = \frac{d^2}{2D}$.

Problem 14. The arrangement used by Thomas Young to produce an interference pattern is shown in Fig. 10.57. Justify why there would be no change in the 'fringe width' when the main illuminated slit (S) is shifted to the position S′ as shown. [CBSE SP 15]

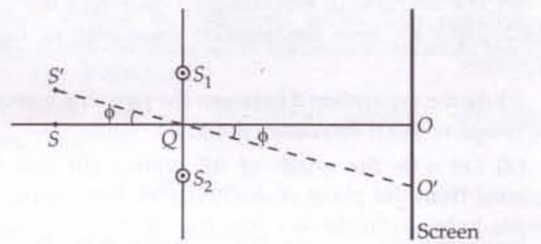


Fig. 10.57

Solution. If the source S is on the perpendicular bisector of S_1S_2, then the central fringe O also lies on this perpendicular bisector. If S is shifted by an angle ϕ to a new point S', then the central fringe appears at a point O' at an angle $-\phi$, which means it is shifted by the same angle on the other side of the bisector. Thus, the source S', midpoint Q of S_1S_2 and the central fringe O' all lie on a straight line. Only the fringe pattern is shifted by amount OO', whereas the fringe width $\beta = \frac{D\lambda}{d}$ remains the same.

Problem 15. One of the two slits in Young's double slit experiment is so painted that it transmits half the intensity of the other. What is the effect on interference fringes ?

Solution. Let I_0 be the intensity of light from each slit.

When the slit is not painted,

$$I_{max} = (\sqrt{I_0} + \sqrt{I_0})^2 = 4I_0$$

$$I_{min} = (\sqrt{I_0} - \sqrt{I_0})^2 = 0.$$

When one of the slits is painted, it transmits half of the original intensity.

$$\therefore \quad I_{max} = \left(\sqrt{I_0} + \sqrt{\frac{I_0}{2}}\right)^2 = I_0\left(1 + \frac{1}{\sqrt{2}}\right)^2 = 2.914\, I_0$$

$$I_{min} = \left(I_0 - \sqrt{\frac{I_0}{2}}\right)^2 = I_0\left(1 - \frac{1}{\sqrt{2}}\right)^2 = 0.086\, I_0$$

Hence on painting one of the two slits, the intensity of maxima *decreases* from $4I_0$ to 2.914 I_0 and that of minima *increases* from 0 to 0.086 I_0. The contrast between the bright and dark fringes decreases.

Problem 16. Two sources of intensity I_1 and I_2 undergo interference in Young's double slit experiment. Show that $\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$, where a_1 and a_2 are the amplitudes of disturbance for two sources S_1 and S_2. [CBSE D 01C]

Solution. When two light waves of amplitudes a_1 and a_2 and having phase difference ϕ interfere, the resultant amplitude is

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi}$$

As intensity $\propto$ (amplitude)^2

$$\therefore \quad I = ka^2 = k(a_1^2 + a_2^2 + 2a_1a_2\cos\phi)$$

When $\phi = 0$, $\cos\phi = 1$ and the intensity is maximum,

$$I_{max} = k(a_1^2 + a_2^2 + 2a_1a_2 \times 1) = k(a_1 + a_2)^2$$

When $\phi = \pi$, $\cos\phi = -1$, the intensity is minimum,

$$I_{min} = k(a_1^2 + a_2^2 - 2a_1a_2) = k(a_1 - a_2)^2$$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{k(a_1 + a_2)^2}{k(a_1 - a_2)^2} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2.$$

Problem 17. In Fig. 10.58, two light waves of the same frequency start from the sources S_1 and S_2 in the same phase. The distance $S_1S_2 = \lambda/2$. What will be the nature of the interference at the points *A*, *B* and *C* ?

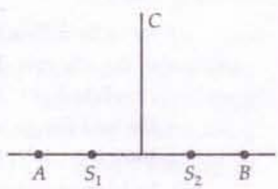


Fig. 10.58

Solution. Both at points *A* and *B*, the path difference between the waves coming from S_1 and S_2 will be $\lambda/2$. Hence destructive interference occurs at *A* and *B*. As $S_1C = S_2C$, the path difference between the waves reaching the point C from S_1 and S_2 will be zero. Hence constructive interference occurs at point C.

Problem 18. Interference can be observed with two independent tuning forks but it cannot be observed with two independent bulbs. Why ?

Solution. When two tuning forks are struck simultaneously they produce sound waves almost in the same phase. Their phase difference, if any, varies slowly with time. Interference pattern also varies slowly with time. Such variations can be detected easily by the human ear. So interference pattern is easily observable.

The phase difference between two independent light bulbs changes 10^8 times per second. The interference pattern also changes 10^8 times per second. Such rapid variations cannot be detected by our eyes. So interference pattern is not observable.

Problem 19. Two narrow slits are illuminated by a single monochromatic source. Name the pattern obtained on the screen. One of the slits is now completely covered. What is the name of the pattern now obtained on the screen ? Draw intensity pattern obtained in the two cases. Also write two differences between the patterns obtained in the above two cases. [OD 04, 06]

Solution. With two narrow slits, an interference pattern is obtained.

When one slit is completely covered, diffraction pattern is obtained.

For intensity distribution curve for interference, see Fig. 10.15 on page 10.21.

For intensity distribution curve for diffraction, see Fig. 10.22 on page 10.31.

Interference	Diffraction
1. All the bright fringes are of same intensity.	Intensity of bright fringes decreases with the increasing order.
2. All the bright fringes are of equal width.	Central bright fringe is twice as wide as any secondary bright fringe.
3. Regions of dark fringes are perfectly dark.	Regions of dark fringes are not perfectly dark.
4. Maxima occur at $\theta = n\frac{\lambda}{d}$	Minima occur at $\theta = n\frac{\lambda}{a}$

Problem 20. What change will occur in diffraction pattern if

(i) light of smaller wavelength is used,

(ii) slit is made narrower, and

(iii) another slit is placed near and parallel to the first slit ?

Solution. Directions of various minima in a diffraction pattern are given by $\theta_n = \frac{n\lambda}{a}$

(i) When light of smaller wavelength λ is used, the diffraction pattern becomes narrower.

(*ii*) When a decreases, θ_n increases and the diffraction pattern spreads out.

(*iii*) Interference pattern replaces the diffraction pattern.

Problem 21. Draw the diagram showing intensity distribution of light on the screen for diffraction of light at a single slit. How is the width of central maxima affected if (*i*) the width of the slit is doubled ; (*ii*) the wavelength of the light used is increased ?

What happens to the width of the central maxima if the whole apparatus is immersed in water and why ?

[CBSE F 09]

Solution. For intensity distribution of light in diffraction at a signal slit, see Fig. 10.22 on page 10.31.

Width of central maximum is given by

$$\beta_0 = \frac{2D\lambda}{a}$$

(*i*) When wavelength (λ) of light used is increased, the width of central maximum increases.

(*ii*) When width (a) of the slit is increased, the width of central maximum decreases.

Wavelength of light in water decreases, so width of central maximum also decreases.

Problem 22. State with reason, how would the linear width of central maximum change if (*i*) monochromatic yellow light is replaced with red light, and (*ii*) distance between the slit and the screen is increased.

[CBSE OD 03C]

Solution. The linear width of central bright maximum is given by

$$\beta_0 = \frac{2D\lambda}{a}$$

(*i*) If monochromatic yellow light is replaced with red light, the linear width of the central maximum increases because $\lambda_{red} > \lambda_{yellow}$.

(*ii*) If the distance (D) between the slit and the screen is increased, the linear width of the central maximum increases.

Problem 23. In a single slit diffraction pattern, how does the angular width of central maximum change, when (*i*) slit width is decreased, (*ii*) distance between the slit and screen is increased and (*iii*) light of smaller visible wavelength is used ? Justify your answer in each case. [CBSE D 94 ; OD 02, 07C]

Solution. Linear width of central maximum,

$$\beta_0 = \frac{2D\lambda}{a}$$

Angular width of central maximum $= \frac{\beta_0}{D} = \frac{2\lambda}{a}$

(*i*) When slit width a decreases, angular width increases.

(*ii*) When distance D between the slit and screen is increased, angular width does not change.

(*iii*) When light of smaller wavelength λ is used, angular width decreases.

Problem 24. State the condition for diffraction of light to occur. In the diffraction at a single slit experiment, how would the width and the intensity of central maximum change, if (*i*) slit width is halved and (*ii*) visible light of longer wavelength is used ? [CBSE D 01]

Solution. Diffraction of light is highly pronounced if the size of the obstacle or aperture is comparable to the wavelength of the light used.

Width of central maximum $= \frac{2D\lambda}{a}$

(*i*) If slit width a is halved, width of central maximum is doubled. Its area becomes 4 times and hence intensity becomes one-fourth of the initial intensity.

(*ii*) If visible light of longer wavelength is used, width of central maximum increases and hence intensity decreases.

Problem 25. In a single slit diffraction experiment, the slit width is made double that of the original width. What would happen to the size and intensity of central diffraction band ? Give reason for your answer.

[CBSE OD 08 ; F12 ; D 12]

Solution. Width of central maximum $= \frac{2D\lambda}{a}$

When slit width (a) is doubled, width of central maximum is halved. Its area becomes 1/ 4 times and hence intensity becomes 4 times the initial intensity.

Problem 26. What two main changes in diffraction pattern of single slit will you observe when the monochromatic source of light is replaced by a source of white light ? [CBSE F 13]

Solution. When the monochromatic source is replaced by a source of white light, the diffraction pattern shows following changes :

(*i*) In each diffraction order, the diffracted image of the slit gets dispersed into component colours of white light. As fringe width $\propto$ wavelength, so the red fringe with higher wavelength is wider than the violet fringe with smaller wavelength.

(*ii*) In higher order spectra, the dispersion is more and it causes overlapping of different colours.

Problem 27. Give reasons for the following :

(*i*) **Astronomers prefer to use telescopes with large objective diameters to observe astronomical objects.**

(*ii*) **Two identical but independent monochromatic sources of light cannot be coherent.**

(*iii*) **The value of the Brewster angle for a transparent medium is different for lights of different colours.** [CBSE Sample Paper 08]

Solution. (*i*) The objective of large aperture has a large light gathering capacity and it forms bright images of even distant faint stars. It also increases the resolving power of the telescope.

(*ii*) Light wave emitted by an ordinary source (such as a sodium lamp) undergoes abrupt phase changes of the order of 10^{-8} s. Thus the light waves coming out from two independent sources of light will not have any fixed phase relationship and would be incoherent.

(*iii*) According to Brewster law, $\mu = \tan i_p$

As refractive index μ of a transparent medium is different for light of different colours, so Brewster angle i_p is different for light of different colours.

Problem 28. How will the angular separation and visibility of fringes in Young's double slit experiment change when (*i*) screen is moved away from the plane of the slits, and (*ii*) width of the source slit is increased ? [CBSE OD 08]

Solution. Fringe width, $\beta = \frac{D\lambda}{d}$

Angular separation, $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$

(*i*) When the screen is moved away from the slits, the distance D increases. Fringe width β increases but angular separation $\theta (= \lambda / d)$ remains unchanged.

(*ii*) The interference pattern becomes less and less sharp. When the source slit becomes so wide that the condition $\frac{s}{S} < \frac{\lambda}{d}$ is not satisfied, the interference pattern disappears. But the angular width $\theta (= \lambda / d)$ remains unchanged.

Problem 29. Distinguish between magnifying power and resolving power of a microscope. [CBSE D 92]

Solution. The *magnifying power of a microscope* is defined as the ratio of the angle subtended by the image at the eye and the angle subtended by the object seen directly, when both lie at the least distance of distinct vision. The magnifying power of a compound microscope is given by

$$M = \frac{v}{u}\left(1 + \frac{D}{f_e}\right) = \left(1 - \frac{v}{f_0}\right)\left(1 + \frac{D}{f_e}\right)$$

where u and v are the distances of object and image from the objective lens.

The resolving power of a microscope is defined as the reciprocal of the minimum distance (d) between two point objects, which can just be seen through the microscope as separate.

$$\text{R.P.} = \frac{1}{d} = \frac{2\mu \sin \theta}{\lambda}$$

where μ is the refractive index of the medium between object and objective lens, θ is half the angle of cone of light from the point object.

Problem 30. Define the terms magnifying power and resolving power of a telescope. [CBSE D 93C]

Solution. The *magnifying power of a telescope* in normal adjustment is defined as the ratio of the angle subtended by the image at the eye as seen through the telescope to the angle subtended by the object directly, when both the object and the image lie at infinity.

In normal adjustment, $m = \frac{f_0}{f_e}$

When the final image is formed at the least distance of distinct vision,

$$m = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$$

The *resolving power of a telescope* is defined as the reciprocal of the smallest angular separation ($d\theta$) between two distant objects, whose images are just seen in the telescope as separate.

$$\text{R.P.} = \frac{1}{d\theta} = \frac{D}{1.22\,\lambda}$$

where D is the diameter of the objective lens.

Problem 31. How does the resolving power of a compound microscope change, when (*i*) refractive index of the medium between the object and the objective lens increases ; and (*ii*) wavelength of the radiation used is increased ? [CBSE OD 05]

Solution. R.P. of a compound microscope

$$= \frac{2\mu \sin \theta}{\lambda}$$

(*i*) When the refractive index (μ) of the medium between the object and the objective lens increases, the resolving power increases.

(*ii*) When the wavelength (λ) of the radiation used is increased, the resolving power decreases.

Problem 32. Explain with reason, how the resolving power of a compound microscope will change when (*i*) frequency of the incident light on the objective lens is increased, (*ii*) focal length of the objective lens is increased, and (*iii*) aperture of the objective lens is increased. [CBSE OD 97, 02]

Solution. R.P. of a compound microscope

$$= \frac{2\mu \sin \theta}{\lambda} = \frac{2\mu \sin \theta \times \nu}{c}$$

(*i*) When the frequency ν of the incident light increases, the resolving power increases (R.P. $\propto \nu$).

(*ii*) Resolving power does not change with change in focal length of objective lens.

(*iii*) When the aperture of the objective lens increases, the semivertical angle θ increases and hence the resolving power of the microscope increases.

Problem 33. **How does the resolving power of a compound microscope change on (*i*) decreasing the wavelength of light used, and (*ii*) decreasing the diameter of its objective lens ?**

[CBSE OD 98 ; Sample Paper 08]

Solution. R.P. of a compound microscope

$$= \frac{2\mu \sin\theta}{\lambda}$$

(*i*) On decreasing the wavelength (λ) of the light used, the resolving power increases.

(*ii*) On decreasing the diameter of the objective lens, semivertical angle θ decreases, so the resolving power also decreases.

Problem 34. **How does the resolving power of a telescope change if (*i*) the size of the aperture of the objective lens is increased, (*ii*) the focal length of the objective lens is decreased ?** [CBSE D 03C]

Solution. Resolving power of a telescope $= \dfrac{D}{1.22\,\lambda}$

(*i*) If the size of the aperture (D) of the objective lens is increased, the resolving power of the telescope increases.

(*ii*) If the focal length of the objective lens is decreased, the resolving power of the telescope is not affected.

Problem 35. **Explain with reason, how the resolving power of an astronomical telescope will change when (*i*) frequency of the incident light on the objective lens is increased, (*ii*) focal length of the objective lens is increased, and (*iii*) aperture of the objective lens is halved.** [CBSE OD 02]

Solution. R.P. of an astronomical telescope

$$= \frac{D}{1.22\,\lambda} = \frac{D\nu}{1.22\,c}$$

(*i*) When frequency of the incident light is increased, the resolving power of the telescope increases.

(*ii*) When the focal length of the objective lens is increased, the resolving power does not change.

(*iii*) When aperture of the objective lens is halved, the resolving power of the telescope is also halved.

Problem 36. **On what factors, does the (*i*) magnifying power and (*ii*) resolving power of a compound microscope depend ?** [CBSE D 98]

Solution. (*i*) Magnifying power of a compound microscope,

$$m = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$$

Clearly, the magnifying power of a compound microscope depends on the focal lengths of its objective and eyepiece.

(*ii*) R.P. of a compound microscope $= \dfrac{2\mu \sin\theta}{\lambda}$

Clearly, the resolving power of a compound microscope depends on (*a*) wavelength of light used to illuminate the object, (*b*) refractive index of the medium between the object and the objective, and (*c*) the semivertical angle θ of the cone of light from the object to the objective.

Problem 37. **On what factors, does the (*i*) magnifying power and (*ii*) resolving power of a refracting type astronomical telescope depend ?** [CBSE D 98]

Solution. (*i*) Magnifying power of a telescope,

$$m = \frac{f_0}{f_e}$$

Clearly the magnifying power of a telescope depends on the focal lengths of its objective and eyepiece.

(*ii*) R.P. of a telescope $= \dfrac{D}{1.22\,\lambda}$

Clearly, the resolving power of a telescope depends on the aperture (D) of its objective and wavelength (λ) of the light used.

Problem 38. **How does the (*i*) magnifying power and (*ii*) resolving power of a telescope change on increasing the diameter of its objective ? Give reasons for your answer.** [CBSE OD 98]

Solution. (*i*) Magnifying power $= \dfrac{f_0}{f_e}$. Clearly, it does not change on increasing the diameter of the objective.

(*ii*) Resolving power of a telescope $= \dfrac{D}{1.22\,\lambda}$

Clearly, resolving power of the telescope increases on increasing the diameter of its objective.

Problem 39. **Which of the following waves can be polarised : (*i*) X-rays, (*ii*) sound waves ? Give reasons.**

[CBSE D 03C]

Solution. (*i*) X-rays can be polarised as these are transverse waves.

(*ii*) Sound waves cannot be polarised as these are longitudinal waves.

Problem 40. **What is a polaroid ? How is plane polarised light obtained with its help ? How will you use it to distinguish between unpolarised light and plane polarised light ?** [CBSE OD 01C ; D 13C]

Solution. A polaroid is a thin commercial sheet which makes use of the property of selective absorption to produce an intense beam of plane polarised light.

When unpolarised light falls on a polaroid, only the vibrations parallel to the transmission plane get transmitted and perpendicular vibrations are selectively absorbed. So the emergent light is plane polarised.

When unpolarised light is seen through the polaroid, the intensity of light is cut down to half due to polarisation of light. When the polaroid is rotated, the intensity of light does not change.

When linearly polarised light is seen through a polaroid and the polaroid is rotated, the transmitted light shows rise and fall of intensity. The intensity becomes twice maximum and twice zero in each rotation.

Problem 41. When a sheet of transparent plastic is placed between two crossed polarizers, no light is transmitted. When the sheet is stretched in one direction, some light passes through the crossed polarizers. What is happening ? [NCERT]

Solution. A transparent plastic sheet is not a polaroid. So when two polaroids are placed with crossed axes, no light is transmitted, whether the plastic sheet is placed between them or not. But when the sheet is stretched, the polymer molecules in it make it a polaroid with its own polaroid axis, which may make some angle with the axes of the two polaroids. Now it becomes a case of three polaroids with the middle polaroid having its axis between the axes of the two fixed polaroids. That is why some light is transmitted in this case.

Problem 42. Show that when a ray of light is incident on the surface of a transparent medium at the polarising angle, the reflected and transmitted rays are perpendicular to each other. [Karnataka 93, 95 ; ISCE 95]

Solution. From Snell's law, $\frac{\sin i_p}{\sin r_p} = \mu$

From Brewster law, $\tan i_p = \frac{\sin i_p}{\cos i_p} = \mu$

$$\therefore \quad \frac{\sin i_p}{\sin r_p} = \frac{\sin i_p}{\cos i_p} \quad \text{or} \quad \sin r_p = \cos i_p$$

or $$\sin r_p = \sin(90° - i_p)$$

$$\therefore \quad r_p = 90° - i_p \quad \text{or} \quad i_p + r_p = 90°$$

Hence the reflected and transmitted rays are perpendicular to each other.

Problem 43. The critical angle between a given transparent medium and air is denoted by i_c. A ray of light in air medium enters this transparent medium at an angle of incidence equal to the polarizing angle (i_p). Deduce a relation for the angle of refraction (r_p) in terms of i_c. [ISCE 98]

Solution. According to Brewster law, when a ray of light is incident on a transparent refracting medium at polarising angle,

$$\mu = \tan i_p$$

But $i_p + r_p = 90°$ or $i_p = 90° - r_p$

$$\therefore \quad \mu = \tan(90° - r_p) = \cot r_p = \frac{1}{\tan r_p} \quad ...(i)$$

As i_c is the critical angle for the transparent medium, so

$$\mu = \frac{1}{\sin i_c} \quad ...(ii)$$

On comparing (*i*) and (*ii*), we get

$$\tan r_p = \sin i_c \quad \text{or} \quad r_p = \tan^{-1}(\sin i_c).$$

Problem 44. Define critical angle and polarising angle. What is the relation between the two angles ? [Punjab 96C, 99, 99C]

Solution. *Critical angle* (i_c). The angle of incidence in the denser medium for which the angle of refraction in the rarer medium is 90° is called critical angle of the denser medium.

Polarising angle (i_p). The angle of incidence, at which when unpolarised light is incident on a transparent refracting medium, the reflected light becomes completely plane polarised, is called the polarising angle.

The relation between the two angles is

$$\tan i_p = \frac{1}{\sin i_c}.$$

Problem 45. Green light is incident at the polarizing angle on a certain glass plate. The angle of refraction is 32°. What are :

(*i*) the polarizing angle,

(*ii*) the index of refraction of glass ?

(*iii*) Indicate the polarisation components (of electric field) on the reflected and refracted rays, by double arrows and dots. [ISCE 03]

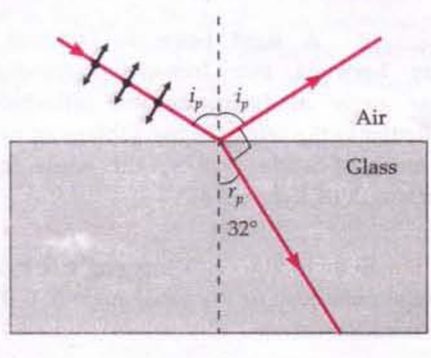


Fig. 10.59

Solution. (*i*) Refracting angle, $r_p = 32°$
Polarising angle,

$$i_p = 90° - r_p = 90° - 32° = 58°.$$

(*ii*) Refractive index of glass,

$$\mu = \tan i_p = \tan 58° = \mathbf{1.6}.$$

(*iii*) The polarisation components of electric field on the reflected and refracted rays are shown in Fig. 10.60.

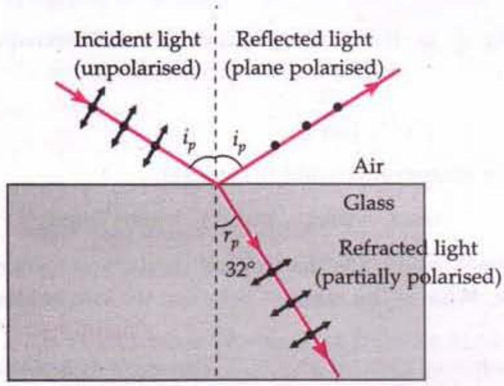


Fig. 10.60

Problem 46. **Define Brewster angle. Show that the Brewster angle 'i_B' for a given pair of transparent media, is related to the critical angle i_c through the relation, $i_c = \sin^{-1}(\cot i_B)$.** [CBSE OD 08C]

Solution. Brewster angle is the angle of incidence at which a beam of unpolarised light falling on a transparent surface is reflected as a beam of completely plane polarised light.

According to Brewster law,

$$\mu = \tan i_B = \frac{1}{\cot i_B}$$

Also, $$\mu = \frac{1}{\sin i_C}$$

On comparing, $\sin i_C = \cot i_B$

Hence, $i_C = \sin^{-1}(\cot i_B)$.

Problem 47. **A light beam is incident on the boundary between two transparent media. At a particular angle of incidence the reflected ray is perpendicular to the refracted ray. Obtain an expression for this angle of incidence. Does this angle depend on the wavelength of light used ?**

[CBSE Sample Paper 11]

Solution. Refer to Fig. 10.60. The angle of incidence is equal to the polarising or Brewster angle (i_p). If r_p is the angle of refraction, then

$$i_p + r_p = 90° \quad \text{or} \quad r_p = 90° - i_p$$

$$\therefore \quad \mu = \frac{\sin i_p}{\sin r_p} = \frac{\sin i_p}{\sin(90° - i_p)}$$

$$= \frac{\sin i_p}{\cos i_p} = \tan i_p$$

$$\therefore \quad i_p = \tan^{-1}(\mu)$$

Yes, i_p depends on wavelength of light used because $\mu \propto \frac{1}{\lambda}$.

Problem 48. **(*a*) Name the phenomenon which proves transverse wave nature of light. Give two uses of the devices whose functioning is based on this phenomenon.**

(*b*) Name the phenomenon which is responsible for bending of light around sharp corners of an obstacle. Under what conditions does this phenomenon take place ? Give one application of this phenomenon in everyday life. [CBSE SP 15]

Solution. (*a*) Polarisation of light proves the transverse nature of light.

Uses. Polaroids are used in sunglasses, window panes, photographic cameras, 3D movies cameras, etc.

(*b*) Diffraction causes bending of light around the corners of small obstacles. The size of the obstacle should be comparable to the wavelength of light incident on it.

Ultrasound scanning uses the principle of diffraction to assess the size and shape of ulcers, tumours, etc., in human body.

Problem 49. **Give reason for each of the following observations :**

(*i*) **The resultant intensity at any point on the screen varies between zero and four times the intensity, due to one slit, in young's double slit experiment.**

(*ii*) **A few coloured fringes, around a central white region, are observed on the screen, when the source of monochromatic light is replaced by white light in Young's double slit experiment.**

(*iii*) **The intensity of light transmitted by a polariod is half the intensity of the light incident on it.**

[CBSE Sample Paper 11]

Solution. (*i*) The resultant intensity, at any point on the screen, is given by

$$I = 4I_0 \cos^2 \frac{\phi}{2}$$

For constructive interference :

$$\phi = 0, 2\pi, 4\pi \text{ and so on}$$

$\Rightarrow \quad I = 0$ for minimum intensity.

For destructive interference :

$$\phi = \pi, 3\pi, 5\pi \text{ and so on}$$

$\Rightarrow \quad I = 4I_0$ for maximum intensity.

Thus, intensity varies between zero and four times the intensity, due to each slit, in Young's double slit experiment.

(*ii*) The interference patterns due to different colours of white light overlap incoherently. The central bright fringes for different colours are at the same position. Therefore the central fringe is white and the fringes closest, on either side of central white fringe, are red and the farthest will appear blue. After a few fringes no clear fringe pattern is seen.

(*iii*) A polaroid consists of a long chain of molecules aligned in a particular direction. The electric vector (associated with the propagating light wave) along the direction of the aligned molecules get absorbed. Thus, if the light, from an ordinary source, passes through a polaroid, it is observed that its transmitted intensity gets reduced by half.

HOTS Problems on Higher Order Thinking Skills

Problem 1. *A ray of light of frequency 5×10^{14} Hz is passed through a liquid. The wavelength of light measured inside the liquid is found to be 450 nm. Calculate (i) wavelength of light in vacuum (ii) refractive index of liquid (iii) velocity of light in the liquid. Take velocity of light in vacuum as 3×10^8 ms^{-1}.*

Solution. Here $\nu = 5 \times 10^{14}$ Hz,

$$\lambda_l = 450 \text{ nm} = 450 \times 10^{-9} \text{m}$$

Velocity in liquid,

$$v_l = \nu \lambda_l = \mathbf{2.25 \times 10^8 \text{ ms}^{-1}}.$$

Refractive index, $\mu = \dfrac{c}{v_l} = \dfrac{3 \times 10^8}{2.25 \times 10^8} = \mathbf{1.33}$

Wavelength in vacuum,

$$\lambda = \mu \lambda_l = 1.33 \times 450 \text{ nm} = \mathbf{600 \text{ nm}}.$$

Problem 2. *A plane wavefront, of width x, is incident on an air-water interface and the corresponding refracted wavefront has a width z as shown. Express the refractive index of air with respect to water, in terms of the dimension shown.* **[CBSE Sample Paper 11]**

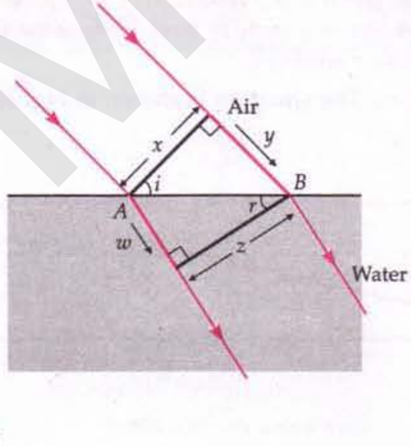


Fig. 10.61

Solution. $^a\mu_w = \dfrac{\sin i}{\sin r} = \dfrac{y / AB}{w / AB} = \dfrac{y}{w}$

$$\therefore \quad ^w\mu_a = \frac{w}{y}.$$

Problem 3. *Find the maximum intensity in case of interference of n identical waves each of intensity I_0, if the interference is (i) coherent and (ii) incoherent.* **[IIT]**

Solution. (*i*) ***When the interference is coherent.*** When two waves of intensities I_1 and I_2 and having a phase difference ϕ interfere, the resultant intensity at any point is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

For intensity to be maximum,

$$\phi = 0 \quad \text{or} \quad \cos \phi = 1$$

$$\therefore \quad I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$$

When n identical waves of each of intensity I_0 interfere,

$$I_{max} = (\sqrt{I_0} + \sqrt{I_0} + \sqrt{I_0} + \ldots\ldots n \text{ terms})^2 = (n\sqrt{I_0})^2$$

or $\quad I_{max} = \mathbf{n^2 I_0}$.

(*ii*) ***When interference is incoherent.*** Here the phase difference ϕ between two waves changes randomly with time. So the average value of $\cos\phi$ over a complete cycle is zero.

Consequently,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \times 0 = I_1 + I_2$$

When n identical waves each of intensity I_0 interfere,

$$I_{max} = I_0 + I_0 + I_0 + \ldots\ldots n \text{ terms} = \mathbf{n\, I_0}$$

Problem 4. *The intensity, at the central maximum (O) in a Young's double slit setup shown in Fig. 10.62 is I_0. If the distance OP equals one third of the fringe width of the pattern, show that the intensity, at point P, would equal $I_0 / 4$.* **[CBSE Sample Paper 11]**

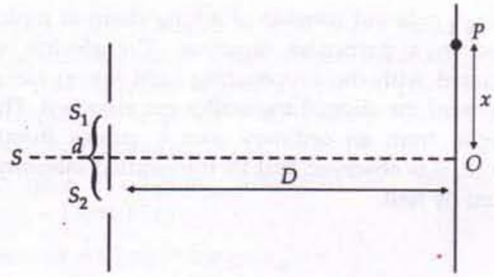


Fig. 10.62

Solution. Given $x = \frac{1}{3}\beta = \frac{1}{3}.\frac{D\lambda}{d}$

Path difference, $p = \frac{xd}{D} = \frac{\lambda}{3}$

Phase difference, $\phi = \frac{2\pi p}{\lambda} = \frac{2\pi}{3}$

Hence, intensity at point P would be

$$I = I_0 \cos^2 \frac{\phi}{2} = I_0 \cos^2 \frac{\pi}{3} = \frac{I_0}{4}.$$

Problem 5. *Figure 10.63 shows a modified Young's double slit experimental set up. Here $SS_2 - SS_1 = \lambda/4$.*

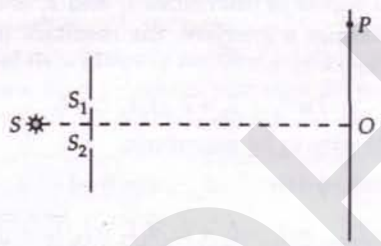


Fig. 10.63

(i) State the condition for constructive and destructive interference.

(ii) Obtain an expression for the fringe width.

(iii) Locate the position of the central fringe.

[CBSE OD 13C]

Solution.

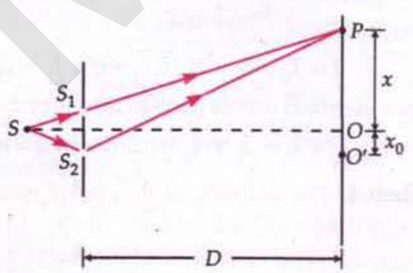


Fig. 10.64

Initial path difference between S_1 and S_2,

$$\Delta_0 = SS_2 - SS_1 = \frac{\lambda}{4}$$

Path difference between disturbances from S_1 and S_2 at point P,

$$\Delta = \frac{xd}{D}$$

Total path difference between the two disturbances at P,

$$\Delta_T = \Delta_0 + \Delta = \frac{\lambda}{4} + \frac{xd}{D}$$

∴ For constructive interference,

$$\Delta_T = \left(\frac{\lambda}{4} + \frac{xd}{D}\right) = n\lambda\,;\ n = 0, 1, 2, ...$$

or $$\frac{x_n d}{D} = \left(n - \frac{1}{4}\right)\lambda \qquad ...(1)$$

For destructive interference,

$$\Delta_T = \left(\frac{\lambda}{4} + \frac{xd}{D}\right) = (2n-1)\frac{\lambda}{2} \qquad ...(2)$$

or $$\frac{x'_n d}{D} = \left(2n - 1 - \frac{1}{2}\right)\frac{\lambda}{2}$$

or $$\frac{x'_n d}{D} = \left(2n - \frac{3}{2}\right)\frac{\lambda}{2}$$

Fringe width, $\beta = x_{n+1} - x_n = \frac{\lambda D}{d}$

The position x_0 of central fringe is obtained by putting $n = 0$ in equation (1). Therefore,

∴ $$x_0 = -\frac{\lambda D}{4d}$$

The negative sign shows that the central fringe is obtained at a point O' below the (central) point O.

Problem 6. *White light is used to illuminate two slits in Young's double slit experiment. The separation between the slits is b, and the screen is at a distance d(>> b) from the slits. At a point on the screen directly in front of one of the slits, which wavelengths are missing ?* **[IIT]**

Solution. The situation is shown in Fig. 10.65.

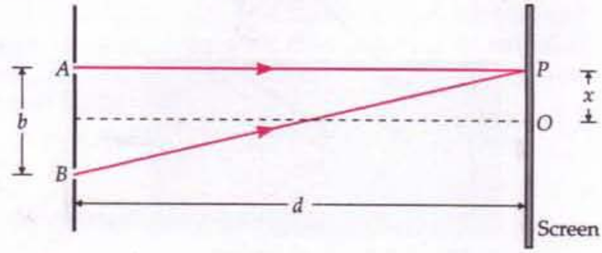


Fig. 10.65

The path difference between the waves reaching the point P is

$$p = BP - AP$$

$$= (d^2 + b^2)^{1/2} - d = d\left(1 + \frac{b^2}{d^2}\right)^{1/2} - d$$

$$\approx d\left(1 + \frac{1}{2}\frac{b^2}{d^2}\right) - d = \frac{b^2}{2d}$$

For a dark band,

$$p = \frac{b^2}{2d} = (2n-1)\frac{\lambda}{2},$$

where $n = 1, 2, 3, \ldots$

or $$\lambda = \frac{1}{(2n-1)} \cdot \frac{b^2}{d}$$

Hence the missing wavelengths are : $\frac{b^2}{d}, \frac{b^2}{3d}, \frac{b^2}{5d}, \ldots$

Problem 7. *Light, from a monochromatic source, is made to fall on a single slit of variable width. An experimentalist records the following data for the linear width of the principal maxima on a screen kept at a distance of 1 m from the plane of the slit.*

S.No.	Width of Slit	Linear width of principal maxima
1	0.1 mm	6 mm
2	0.2 mm	3 mm
3	0.3 mm	1.98 mm
4	0.4 mm	1.5 mm
5	0.5 mm	1.2 mm

Use any two observations from this data to estimate the value of the wavelength of light used. **[CBSE OD 08]**

Solution. Linear width of a principal maximum,

$$\beta_0 = \frac{2D\lambda}{a} \qquad \therefore \quad \lambda = \frac{\beta_0 a}{2D}$$

From observation at S.No. 1 :

$$a = 0.1 \text{ mm}, \quad \beta_0 = 6 \text{ mm}, \quad D = 1 \text{ m}$$

$$\therefore \quad \lambda = \frac{6 \times 10^{-3} \times 0.1 \times 10^{-3}}{2 \times 1} = \mathbf{3 \times 10^{-7} m.}$$

From observation at S.No. 2 :

$$a = 0.2 \text{ mm}, \beta_0 = 3 \text{ mm}, \quad D = 1 \text{ m}$$

$$\therefore \quad \lambda = \frac{3 \times 10^{-3} \times 0.2 \times 10^{-3}}{2 \times 1} = \mathbf{3 \times 10^{-7} m.}$$

Problem 8. *In a double slit interference experiment, the two coherent beams have slightly different intensities I and $I + \delta I$ ($\delta I << I$). Show that the resultant intensity at the maxima is nearly $4I$ while that at the minima is nearly $\frac{|\delta I|^2}{4I}$.* **[CBSE Sample Paper 08]**

Solution. For any interference pattern,

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 \quad \text{and} \quad I_{min} = (\sqrt{I_1} \sim \sqrt{I_2})^2$$

Given $I_1 = I$ and $I_2 = I + \delta I$ $\qquad [\delta I << I]$

$$\therefore \quad I_{max} = (\sqrt{I} + \sqrt{I + \delta I})^2$$

$$= I + (I + \delta I) + 2\sqrt{I(I + \delta I)}$$

$$= 2I + \delta I + 2I\sqrt{1 + \frac{\delta I}{I}} = \mathbf{4I}$$

$$I_{min} = (\sqrt{I + \delta I} - \sqrt{I})^2$$

$$= I\left[\left(1 + \frac{\delta I}{I}\right)^{1/2} - 1\right]^2$$

$$= I\left[\left(1 + \frac{\delta I}{2I}\right) - 1\right]^2 = I\left(\frac{\delta I}{2I}\right)^2 = \frac{|\delta \boldsymbol{I}|^2}{\mathbf{4I}}.$$

Problem 9. *Two sources S_1 and S_2 emitting light of wavelength 600 nm are placed 0.1 mm apart. A detector is moved on the line S_1P which is perpendicular to S_1S_2 (i) What would be the minimum and maximum path difference at the detector as it is moved along the line S_1P. (ii) Locate the position of farthest minimum detected.*

Solution. (*i*) The situation is shown in Fig. 10.66. The path difference is minimum when the detector is at large distance from S_1. Then the path difference is near to zero.

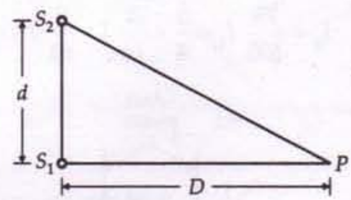


Fig. 10.66

The path difference is maximum when the detector lies at point S_1.

$\therefore$ Maximum path difference $= S_1S_2 = \mathbf{0.1\ mm}$.

(*ii*) The farthest minimum will occur at a point P for which the path difference is $\frac{\lambda}{2}$.

Let $S_1P = D$. Then

$$p = S_2P - S_1P = \frac{\lambda}{2}$$

or $$\sqrt{D^2 + d^2} - D = \frac{\lambda}{2}$$

or $$D^2 + d^2 = \left(D + \frac{\lambda}{2}\right)^2$$

or $$d^2 = D\lambda + \frac{\lambda^2}{4}$$

or $$D = \frac{d^2}{\lambda} - \frac{\lambda}{4} = \frac{(0.1\times10^{-3})^2}{600\times10^{-9}} - \frac{600\times10^{-9}}{4}$$

$$= \frac{1}{60} - 150\times10^{-9} \simeq \frac{1}{60}\text{ m} = \mathbf{1.7\ cm.}$$

Problem 10. *A narrow monochromatic beam of light of intensity I is incident on a glass plate. Another identical glass plate is kept close to the first one and parallel to it. Each plate reflects 25% of the incident light and transmits the remaining. Calculate the ratio of minimum and maximum intensity in the interference pattern formed by two beams obtained after reflection from each plate.*

[CBSE Sample Paper 98 ; IIT 90]

Solution. Let I be the intensity of beam 1 incident on first glass plate. Each plate reflects 25% of light incident on it and transmits 75%.

Therefore,

$$I_1 = I\,;$$

$$I_2 = \frac{25}{100}I = \frac{I}{4}$$

$$I_3 = \frac{75}{100}I = \frac{3}{4}I$$

$$I_4 = \frac{25}{100}I_3 = \frac{1}{4}\times\frac{3}{4}I = \frac{3}{16}I$$

$$I_5 = \frac{75}{100}I_4 = \frac{3}{4}\times\frac{3}{16}I = \frac{9}{64}I$$

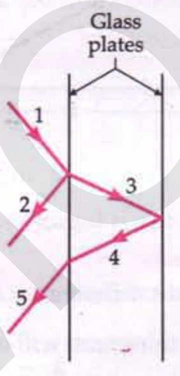


Fig. 10.67

$\therefore$ Amplitude ratio of beams 2 and 5 is

$$r = \sqrt{\frac{I_2}{I_5}} = \sqrt{\frac{I}{4}\times\frac{64}{9I}} = \frac{4}{3}$$

$$\frac{I_{min}}{I_{max}} = \left[\frac{r-1}{r+1}\right]^2 = \left[\frac{\frac{4}{3}-1}{\frac{4}{3}+1}\right]^2 = \frac{1}{49} = \mathbf{1:49.}$$

Problem 11. *A double-slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1.0 mm, and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å. (i) Calculate the fringe-width. (ii) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minimum on the axis.* **[IIT 96]**

Solution. Here $\mu_l = 1.33,\ d = 1.0\text{ mm} = 10^{-3}\text{ m}$
$D = 1.33\text{ m},\ \lambda = 6300\text{ Å},\ \mu_g = 1.53$

(*i*) The wavelength of light in the liquid is

$$\lambda_l = \frac{\lambda}{\mu_l} = \frac{6300\times10^{-10}}{1.33}\text{ m}$$

Fringe width,

$$\beta = \frac{D\lambda_l}{d} = \frac{1.33\times6300\times10^{-10}}{10^{-3}\times1.33}$$

$$= 6.3\times10^{-4}\text{ m} = \mathbf{0.63\ mm.}$$

(*ii*) When one of the slits is covered by a glass sheet of thickness t, the fringe-displacement is given by

$$\Delta x = \frac{D}{d}({}^{l}\mu_g - 1)\,t = \frac{\beta}{\lambda_l}\left(\frac{\mu_g}{\mu_l} - 1\right)t \qquad \left[\because\ \beta = \frac{D\lambda_l}{d}\right]$$

But on covering one of the slits, the adjacent minimum gets shifted to the centre, so the fringe displacement is half the fringe-width *i.e.*, $\Delta x = \beta/2$.

$$\therefore \qquad \frac{\beta}{2} = \frac{\beta}{\lambda_l}\left(\frac{\mu_g}{\mu_l} - 1\right)t$$

or $$t = \frac{\lambda_l}{2\left[\frac{\mu_g}{\mu_l} - 1\right]}$$

$$= \frac{(6300/1.33)\text{ Å}}{2\left[\frac{1.53}{1.33} - 1\right]} = \frac{6300}{2\times0.20}\text{ Å}$$

$$= \mathbf{15750\ Å.}$$

Problem 12. *Figure 10.68 shows an outline of Lloyd's mirror experiment. M is a plane mirror ; S is a narrow slit illuminated by some source of light (not shown) and S' is the image of S in M. M, S and S' are in a plane perpendicular to the paper. O is the line of intersection of the mirror and the screen.*

(*a*) *What is the origin of fringes observed on the screen ?*

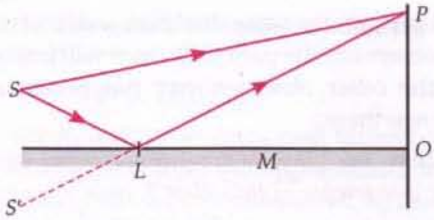


Fig. 10.68

(*b*) *Why is the slit S placed so as to have very oblique angle of incidence of light striking the mirror ?*

(*c*) *The two path lengths PS and PS' are equal when P coincides with O. Yet the fringe at O is found in the experiment to be dark not bright. What does this observation imply ?* **[NCERT]**

Solution. (*a*) S' is the virtual image of source S formed by mirror M. So S and S' act as two coherent sources of light. Light waves coming directly from the source S and the reflected waves (which appear to come from virtual source S') interfere to produce a fringe pattern.

(*b*) Very oblique angle of incidence requires the source S to be placed very close to the mirror. In that case the separation between the coherent sources S and S' will be small, as required in Young's double slit experiment for obtaining broad and distinct interference fringes.

(*c*) The light wave reflected by the mirror suffers a phase change of 180° which is equivalent to a change in the path length of $\lambda/2$. Then the path difference for any point P on the screen becomes

$$p = S'P - SP + \frac{\lambda}{2}$$

Consequently, the condition for a dark fringe is

$$p = S'P - SP + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or $\quad S'P - SP = n\lambda$

This condition is satisfied by the central fringe for which $S'P = SP$. Hence the central fringe in Lloyd's mirror method is dark.

Problem 13. *Figure 10.69 shows two flat glass plates P_1 and P_2 placed nearly (but not exactly) parallel forming an air wedge. The plates are illuminated normally by monochromatic light and viewed from above. Light waves reflected from the upper and lower surfaces of the air wedge give rise to an interference pattern :*

(*a*) *Show that the separation between two successive bright (or dark) fringes is given by $\frac{\lambda l}{2s}$, where l is the length of each plate and s is the separation between the plates at the open end of the wedge.*

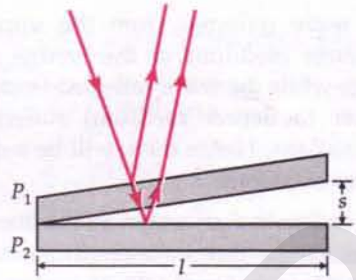


Fig. 10.69 **Geometry of the air wedge interference experiment.**

(*b*) *In the experiment, a dark fringe is observed along the line joining the two plates. Why ?*

(*c*) *If the space between the glass plates is filled with water, what changes in the fringe pattern do you expect to see, if at all ?*

(*d*) *Suggest a way of obtaining a bright fringe along the line of contact of the two plates in this experiment.* **[NCERT]**

Solution. (*a*) Let the separation between the plates at a distance x from the line of contact be y. Then

$$\tan\theta = \frac{y}{x} = \frac{s}{l} \quad \text{or} \quad y = \frac{x \times s}{l}$$

For normal incidence, the path difference between two waves reflected from the upper and lower surfaces of the air wedge at distance x will be twice the thickness of the wedge at that point, *i.e.*,

$$p = 2y = \frac{2xs}{l}$$

The wave reflected from the lower surface suffers an extra path difference of $\frac{\lambda}{2}$, so that

$$p = \frac{2xs}{l} + \frac{\lambda}{2}$$

$\therefore$ Condition for nth dark fringe can be written as

$$p = \frac{2xs}{l} + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or $\quad x = \frac{n\lambda l}{2s}$

The position of $(n-1)$th dark fringe would be

$$x' = \frac{(n-1)\lambda l}{2s}$$

$\therefore$ Fringe width,

$$\beta = x' - x = \frac{\lambda l}{2s}[n-(n-1)]$$

or $\quad \beta = \frac{\lambda l}{2s}$

This proves the required result.

(*b*) The wave reflected from the upper surface (denser to rarer medium) of the wedge suffers no phase change while the wave reflected from the lower surface (rarer to denser medium) suffers a phase change of π radians. Hence there will be a dark fringe along the line of contact.

(*c*) Glass is denser than water. So the line of contact is still a dark fringe. But wavelength in water is less than that in air by a factor of 1.33. Therefore, the fringe width is reduced by this factor.

(*d*) By choosing the upper plate material, medium filling the wedge, and the lower plate material in increasing (or decreasing) order of refractive index, we get a bright fringe along the line of contact.

Problem 14. *Give the shape of interference fringes observed*

(a) in a Young's double-slit experiment

(b) in the air wedge experiment (Problem 13)

(c) in the Lloyd's mirror experiment (Problem 12)

(d) when a small lamp is placed before a thin mica sheet and light waves reflected from the front and back surfaces of the sheet combine to produce interference pattern on a screen behind the lamp. (Pohl's experiment)

(e) from a thin air film formed by placing a convex lens on top of a flat glass plate (Newton's arrangement).

[NCERT]

Solution. (*a*) Interference fringes are *straight lines* parallel to the two slits.

(*b*) Interference fringes are *straight lines* parallel to the edge of the wedge.

(*c*) Interference fringes are *straight lines* parallel to the slit sources.

(*d*) Here the two coherent sources are the point images of the small lamp formed due to the reflection of light waves from the front and back surfaces of the sheet. They form *circular* fringes.

(*e*) Due to circular symmetry, the locii of points of equal thickness are concentric circles. Hence the fringes are concentric circles with the centre at the point of contact of the lens and the plate.

Problem 15. *An observer sees a green fringe passing through a given point in an oil film. Would other observer looking at the same point necessarily see green fringe there ? Explain by writing down the necessary relation.*

Solution. The path difference between the two rays is $2 \mu t \cos r$. Therefore, for given oil film, μ and t are constant and the path difference depends only on angle of refraction, which in turn depends on angle of incidence. It is not possible that two different observers look at a point in the same direction and, therefore, for the two observers, the path difference will not be same. Hence, the other observer may not necessarily see green fringe there.

Problem 16. *Why do we need a broad source for observing interference in thin films ?*

Solution. As shown in Fig. 10.70(*a*), when a point source is used, the rays reflected from the thin film get diverged through wide angles. Due to its small size, the eyelens cannot see the entire interference pattern.

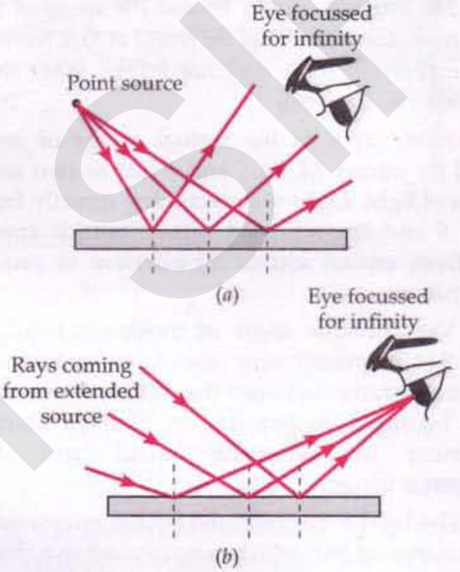


Fig. 10.70

But when an extended source of light is used, as shown in Fig. 10.70(*b*), the rays reflected by the thin film are converging. One can see the entire interference pattern by placing the eye in a suitable position.

Problem 17. *Derive the condition $2\mu t = n\lambda$ for darkness when monochromatic light of wavelength λ is normally incident on a thin oil film of thickness t and refractive index μ.* **[ISCE 03]**

Solution. Fig. 10.71 shows the rays reflected from the upper and lower surfaces of a thin film of thickness t for almost normal incidence.

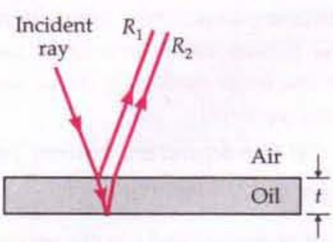


Fig. 10.71

Clearly, the path difference between the reflected rays R_1 and R_2

$$\approx 2t \text{ in oil} = 2\mu t$$

But ray R_1 suffers an extra path difference of $\lambda/2$ due to its reflection from the upper denser surface of oil, so the net path difference between R_1 and R_2 is

$$p = 2\mu t + \frac{\lambda}{2}$$

For a dark fringe,

$$p = 2\mu t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad 2\mu t = n\lambda$$

This is the required condition for a dark fringe.

Problem 18. *A slit of width 'a' is illuminated by white light. For what value of 'a' is the first minimum, for red light of $\lambda = 650$ nm, located at point P ? For what value of the wavelength of light will the first diffraction maxima also fall at P ?* **[CBSE Sample Paper 11]**

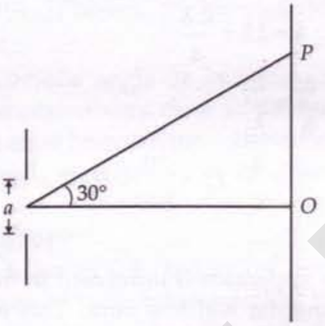


Fig. 10.72

Solution. For first minimum, $n = 1$

$$a \sin 30° = \lambda \quad \text{or} \quad a \times \frac{1}{2} = 650 \text{ nm or } a = \mathbf{1300\ nm}$$

For first maximum to lie at P,

$$a \sin\theta = \frac{3}{2}\lambda' \quad \text{or} \quad \lambda' = \frac{2a\sin\theta}{3}$$

$$= \frac{2 \times 1300 \times \sin 30°}{3} = \mathbf{433.3\ nm.}$$

Problem 19. *The following table gives data about the single slit diffraction experiment :*

Wavelength of light	Half angular width of the principal maxima
λ	θ
$p\lambda$	$q\theta$

Find the ratio of the widths of the slits used in the two cases. Would the ratio of the half angular widths of the first secondary maxima, in the two cases, be also equal to q ? **[CBSE Sample Paper 13]**

Solution. Let a and a' be the slit widths in the two cases. Then

$$\theta = \frac{\lambda}{a} \quad \text{and} \quad q\theta = \frac{p\lambda}{a'}$$

$$\therefore \quad \frac{a'}{a} = \frac{p\lambda / q\theta}{\lambda/\theta} = \frac{p}{q}$$

Yes, the ratio of the half angular widths of the first secondary maxima in the two cases will be equal to q because,

$$\text{required ratio} = \frac{\frac{3}{2}\frac{p\lambda}{a'}}{\frac{3}{2}\frac{\lambda}{a}} = p \times \frac{a}{a'} = p \times \frac{q}{p} = q$$

Problem 20. *Two convex lenses, of equal focal length, but of aperture A_1 and A_2 ($A_2 < A_1$) are used as the objective lenses in two astronomical telescopes having identical eyepieces. Compare the ratio of their (i) resolving power (ii) (normal) magnifying power and (iii) intensity of images formed by them. Which one of the two telescopes should be preferred ? Why ?* **[CBSE D 11]**

Solution. (*i*) R.P. of a telescope $= \dfrac{D}{1.22\lambda}$

$$\Rightarrow \quad \frac{(R.P.)_1}{(R.P.)_2} = \frac{A_1}{A_2} > 1$$

(*ii*) M.P. of a telescope (in normal adjustment) $= \dfrac{f_0}{f_e}$

$$\therefore \quad \frac{(M.P.)_1}{(M.P.)_2} = 1 : 1,$$

because f_0 and f_e are same in both cases.

(*iii*) Intensity ratio of images,

$$\frac{I_1}{I_2} = \frac{A_1}{A_2} > 1$$

The telescope with objective of aperture A_1 should be preferred for viewing as this would

(*a*) give a better resolution.

(*b*) have a higher light gattering power.

Problem 21. *In a pinhole camera, a box of length L has a hole of radius a in one wall. When the hole is illuminated by a parallel beam, the size of the spot of light is large when a is large. Show that it is also very large when a is small, due to diffraction. Assume that the spread due to diffraction just adds to the geometrical spread and find the minimum size of the spot.* **[NCERT]**

Solution. Angular spread due to diffraction $= \dfrac{\lambda}{a}$

$\therefore$ Linear spread on the screen of camera $= \dfrac{L\lambda}{a}$

Thus the size of spot of light becomes large when a is small.

As the incident beam is parallel, the geometrical linear spread will be a.

$\therefore$ Size of the spot

$$= a + \frac{L\lambda}{a}$$

$$= (\sqrt{a})^2 + \left(\sqrt{\frac{L\lambda}{a}}\right)^2 - 2\sqrt{L\lambda} + 2\sqrt{L\lambda}$$

$$= \left(\sqrt{a} - \sqrt{\frac{L\lambda}{a}}\right)^2 + \sqrt{4L\lambda}$$

Hence the size of the spot will be minimum when

$$\sqrt{a} - \sqrt{\frac{L\lambda}{a}} = 0 \quad \text{or} \quad a = \frac{L\lambda}{a}$$

i.e., when geometric and diffraction broadenings are equal.

Minimum size of the spot $= \sqrt{4L\lambda}$.

Problem 22. *In Young's double slit experiment, the distance d between the slits S_1 and S_2 is 1 mm. What should the width of each slit be so as to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern ?* **[NCERT]**

Solution. The linear separation between n bright fringes in an interference pattern on the screen is given by

$$x_n = \frac{n\lambda D}{d}$$

As $x_n << D$, the angular separation between n bright fringes should be

$$\theta_n = \frac{x_n}{D} = \frac{n\lambda}{d}$$

For 10 bright fringes, we get, $\theta_{10} = \frac{10\lambda}{d}$

The angular width of the central maximum in the diffraction pattern due to slit of width a is

$$2\theta_1 = \frac{2\lambda}{a}$$

We want

$$10\frac{\lambda}{d} < 2\frac{\lambda}{a} \quad \text{or} \quad a \le \frac{d}{5} = \frac{1}{5}\text{ mm} = \mathbf{0.2\ mm}.$$

Problem 23. *Angular width of a central maximum in the Fraunhoffer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, the angular width decreases by 30%. Calculate the wavelength of this light. The same decrease in the angular-width of central maximum is obtained when the original apparatus is immersed in a liquid. Find refractive index of the liquid.* **[IIT 96]**

Solution. In single slit diffraction, first minimum occurs at

$$a\sin\theta = \lambda \quad \text{or} \quad \sin\theta = \frac{\lambda}{a}$$

As $\lambda << a$, so $\theta \simeq \sin\theta = \frac{\lambda}{a}$

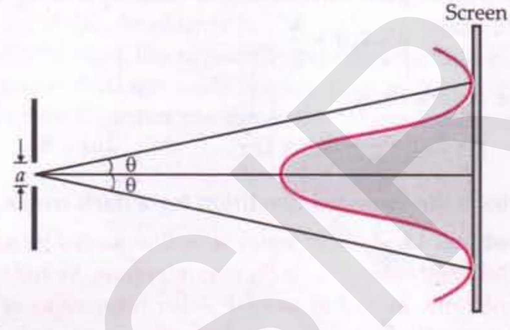


Fig. 10.73

Angular width of central maximum is

$$\phi = 2\theta = \frac{2\lambda}{a}$$

$$\therefore \quad \frac{\phi_2}{\phi_1} = \frac{\lambda_2}{\lambda_1}$$

or

$$\lambda_2 = \frac{\phi_2}{\phi_1}\cdot\lambda_1 = \frac{70}{100} \times 6000 = \mathbf{4200\ Å}$$

$[\because \phi_2 = 70\% \text{ of } \phi_1]$

When the apparatus is immersed in the liquid, the decrease in angular width is same. This indicates that the wavelength of light in the liquid is also 4200 Å.

$$\mu = \frac{\lambda}{\lambda_l} = \frac{6000}{4200} = \mathbf{1.43}.$$

Problem 24. *Find an expression for intensity of transmitted light when a polaroid sheet is rotated between two crossed polaroids. In which position of the polaroid sheet will the transmitted intensity be maximum ?* **[NCERT ; CBSE D 15]**

Solution. Let I_0 be the intensity of polarised light transmitted by first polariser P_1. Then the intensity of light transmitted by second polariser P_2 will be

$$I = I_0\cos^2\theta$$

where θ is the angle between the pass axes of P_1 and P_2. As P_1 and P_3 are crossed, the angle between P_2 and P_3 will be $(\pi/2 - \theta)$. The intensity of light transmitted by P_3 will be

$$I = I_0\cos^2\theta\cos^2\left(\frac{\pi}{2} - \theta\right)$$

$$= I_0\cos^2\theta\sin^2\theta = \frac{I_0}{4}\sin^2 2\theta$$

The transmitted intensity will be maximum when

$$2\theta = \pi/2 \quad \text{or} \quad \theta = \pi/4.$$

Problem 25. *Three identical polaroid sheets P_1, P_2 and P_3 are oriented so that the (pass) axes of P_2 and P_3 are inclined at angles of 60° and 90°, respectively, with respect to the (pass) axis of P_1. A monochromatic source, S, of intensity I_0, is kept in front of the polaroid sheet P_1. Find the intensity of the light, as observed by observers O_1, O_2 and O_3, positioned as shown below.* **[CBSE D 13C]**

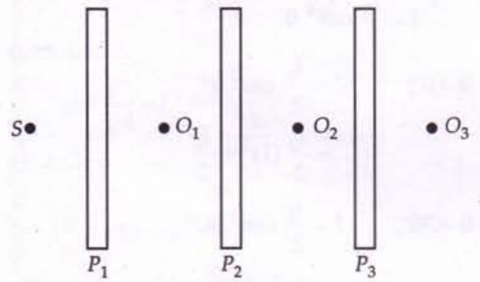


Fig. 10.74

Solution. Intensity observed by observer O_1

$$= \frac{I_0}{2}$$

Intensity observed by O_2

$$= \frac{I_0}{2}\cos^2 60° = \frac{I_0}{2} \times \frac{1}{4} = \frac{I_0}{8}$$

Intensity observed by O_3

$$= \frac{I_0}{8}\cos^2 (90°-60°) = \frac{I_0}{8}\cos^2 30°$$

$$= \frac{I_0}{8} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3I_0}{32}.$$

Problem 26. *What do we understand by 'polarization' of a wave ? How does this phenomenon help us to decide whether a given wave is transverse or longitudinal in nature ?*

Light from an ordinary source (say, a sodium lamp) is passed through a polarised sheet P_1. The transmitted light is then made to pass through a second polaroid sheet P_2 which can be rotated so that the angle (θ) between the two polaroid sheets varies from 0° to 90°. Show graphically the variation of the intensity of light, transmitted by P_1 and P_2, as a function of the angle θ. Take the incident beam intensity as I_0. Why does the light from a clear blue portion of the sky, show a rise and fall of intensity when viewed through a polaroid which is rotated ? **[CBSE Sample Paper 08]**

Solution. The phenomenon of restricting the oscillations of a wave to just one direction in the transverse plane is called polarisation.

As only transverse waves can be polarised and longitudinal waves cannot be polarised, so a wave which exhibits polarisation must have transverse wave nature.

The intensity of light passing through P_1 remains constant equal to $I_0/2$. So I *vs.* θ graph for P_1 is as shown below.

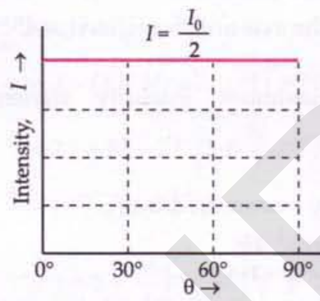


Fig. 10.75 I vs. θ graph for P_1

The intensity of light transmitted by analyser P_2 varies as function of $\cos^2\theta$ as per relation :

$$I = \frac{I_0}{2}\cos^2\theta.$$

So I *vs.* θ graph for P_2 is as shown below.

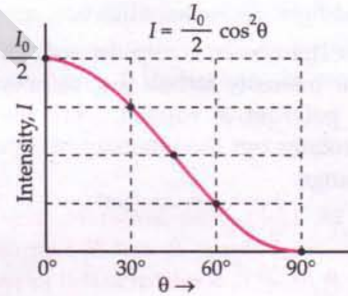


Fig. 10.76 I vs. θ graph for P_2

The light from the blue portion of the sky gets plane polarised due to its scattering by the atmospheric molecules. When seen through a polaroid in a direction perpendicular to direction of incidence, this light shows alternate rise and fall of intensity as the polaroid is rotated.

Problem 27. *(a) What is plane polarised light ? Two polaroids are placed at 90° to each other and the transmitted intensity is zero. What happens when one more polaroid is placed between these two, bisecting the angle between them ? How will the intensity of transmitted light vary on further rotating the third polaroid ?*

(b) If a light beam shows no intensity variation when transmitted through a polaroid which is rotated, does it mean that the light is unpolarised ? Explain briefly. **[CBSE D 08]**

Solution. (*a*) If the electric field vector of a light wave vibrates just in one direction perpendicular to the direction of wave propagation, then the light wave is said to be plane polarised.

Obviously, the axis of P_3 is inclined at 45° to the axis of P_1 and P_2.

Let the maximum intensity transmitted by polariser P_1

$$= I_0$$

The intensity transmitted by P_3

$$= I_0 \cos^2 45°$$
$$= I_0 \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{2}$$

The intensity transmitted by P_2

$$= \frac{I_0}{2} \cos^2 45°$$
$$= \frac{I_0}{2}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{4}$$

Thus the final intensity becomes one-fourth of the maximum intensity transmitted by the first polariod.

On further rotating the third polaroid, the intensity of transmitted light decreases till it becomes zero.

(*b*) Yes, the light beam is unpolarised. The polaroid cuts down the intensity to half due to polarisation of light. When polaroid is rotated, only the plane of polarisation rotates but the intensity of light does not show any change.

Problem 28. *Light, from a sodium lamp, is passed through two polaroid sheets, P_1 and P_2 kept one after the other. Keeping P_1 fixed, P_2 is rotated so that its pass-axis can be at different angles, θ, with respect to the pass-axis of P_1.*

An experimentalist records the following data for the intensity of light coming out of P_2 as a function of the angle θ.

S.No.	θ (Angle between the pass axis of the two polaroids)	*I* (Intensity of light coming out of P_2)
1	0°	$\frac{I_0}{2}$
2	30°	$\frac{3}{8} I_0$
3	45°	$\frac{1}{2\sqrt{2}\, I_0}$
4	60°	$\frac{I_0}{8}$
5	90°	0

[Here I_0 = Intensity of beam falling on P_1]

One of these observations is not in agreement with the expected theoritical variation of I. Identify this observation and write the correct expression. **[CBSE OD 08C]**

Solution. Intensity passing through $P_1 = \frac{I_0}{2}$

Theoretical value of intensity through P_2,

$$I = \frac{I_0}{2}\cos^2\theta$$

At $\theta = 0°$, $I = \frac{I_0}{2}\cos^2 0°$
$$= \frac{I_0}{2}(1)^2 = \frac{I_0}{2}$$

At $\theta = 30°$, $I = \frac{I_0}{2}\cos^2 30°$
$$= \frac{I_0}{2}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3I_0}{8}$$

At $\theta = 45°$, $I = \frac{I_0}{2}\cos^2 45°$
$$= \frac{I_0}{2}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{4}$$

At $\theta = 60°$, $I = \frac{I_0}{2}\cos^2 60°$
$$= \frac{I_0}{2}\left(\frac{1}{2}\right)^2 = \frac{I_0}{8}$$

At $\theta = 90°$, $I = \frac{I_0}{2}\cos^2 90°$
$$= \frac{I_0}{2}(0)^2 = 0.$$

Obviously, the observation at S.No. 3 is not in agreement with the expected theoretical value. The correct expression is $\boldsymbol{I = \frac{1}{4} I_0}$.

Example 29. *Unpolarised light of intensity 32 W m^{-2} passes through three polarisers such that the transmission axis of the last polariser is crossed with the first. If the intensity of the emerging light is 3 W m^{-2}, what is the angle between the transmission axes of the first two polarisers ? At what angle will the transmitted intensity be maximum ?*

[Roorkee 95]

Solution. Let P_1, P_2, P_3 be the three polarisers and θ be the angle between the transmission axes of P_1 and P_2. As P_1 and P_3 are crossed, the angle between P_2 and P_3 is 90° – θ.

Let I_0 be the intensity of the *unpolarised* light falling on P_1. Then the intensity of light emerging from P_1 will be

$$I_1 = \frac{1}{2} I_0 .$$

By Malus law, the intensity of light emerging from P_2 is

$$I_2 = I_1 \cos^2\theta = \frac{1}{2} I_0 \cos^2\theta.$$

The intensity of light emerging from P_3 is

$$I_3 = I_2 \cos^2(90° - \theta)$$

$$= \frac{1}{2} I_0 \cos^2\theta \sin^2\theta$$

$$= \frac{1}{8} I_0 \sin^2 2\theta$$

$$\therefore \quad \sin^2 2\theta = \frac{8 I_3}{I_0} = \frac{8 \times 3}{32} = \frac{3}{4}$$

or $\quad \sin 2\theta = \sqrt{3}/2$

or $\quad 2\theta = 60°$ or $\theta = 30°$.

As $\quad I_3 = \frac{1}{8} I_0 \sin^2 2\theta.$

$\therefore$ I_3 will be maximum when

$$\sin^2 2\theta = 1 \text{ (maximum) or } \sin 2\theta = 1 = \sin 90°$$

or $\quad \theta = 45°$.

10.1. *Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of (i) reflected, and (ii) refracted light ? Refractive index of water is 1.33.* [CBSE Sample Paper 98]

Ans. Here $\mu = 1.33$

(*i*) ***For reflected light***. Since the speed of light in a given medium is fixed and the frequency of light does not change when it is reflected from a surface, therefore, its wavelength should also remain unchanged.

∴ Speed of reflected light

= Speed of (incident) light in air

or $c = 3.0 \times 10^8 \, ms^{-1}$

Wave length of reflected light

= Wavelength of incident light

$$\lambda = 589 \text{ nm} = \mathbf{589 \times 10^{-9} \, m}$$

Frequency of reflected light,

$$\nu = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{589 \times 10^{-9}} = \mathbf{5.09 \times 10^{14} \; Hz.}$$

(*ii*) ***For refracted light.*** Frequency ν remains unchanged. Both wavelength and speed get changed.

Speed of light in water,

$$v_\omega = \frac{c}{\mu} = \frac{3.0 \times 10^8}{1.33} = \mathbf{2.26 \times 10^8 \, ms^{-1}}$$

Wavelength of light in water,

$$\lambda_\omega = \frac{v_\omega}{\nu} = \frac{2.26 \times 10^8}{5.09 \times 10^{14}} = 444 \times 10^{-9} \text{m} = \mathbf{444 \; nm.}$$

10.2. *What is the geometrical shape of the wavefront in each of the following cases :*

(*a*) *Light diverging from a point source.*

(*b*) *Light emerging out of a convex lens when a point source is placed at its focus.*

(*c*) *The portion of the wavefront of light from a distant star intercepted by the earth.*

Ans. (*a*) *Spherical wavefront,* because all such points which are equidistant from the point source will lie on a sphere.

(*b*) *Plane wavefront,* because the light emerges from the convex lens as a parallel beam for which the wavefront is plane.

(*c*) *Plane wavefront,* because a small portion of a spherical wavefront of very large radius is nearly planar.

10.3. (*a*) *The refractive index of glass is 1.5. What is the speed of light in glass ? Speed of light in vacuum is* $3.0 \times 10^8 \, ms^{-1}$.

(*b*) *Is the speed of light in glass independent of the colour of light ? If not, which of the two colours (red and violet) travels slower in a glass prism ?*

Ans. (*a*) Here $\mu = 1.5, \;\; c = 3.0 \times 10^8 \, ms^{-1}$

As $\mu = \dfrac{c}{v}$

∴ Speed of light in glass,

$$v = \frac{c}{\mu} = \frac{3.0 \times 10^8}{1.5} = \mathbf{2.0 \times 10^8 \, ms^{-1}}.$$

(*b*) No, speed of light is not independent of the colour (wavelength) of the light. The violet colour travels slower than the red light in a glass prism. This is because $\mu_V > \mu_R$ and $v = c/\mu$.

10.4. *In a Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.*

Ans. Here $d = 0.28\text{ mm} = 0.28 \times 10^{-3}\text{ m}$, $D = 1.4\text{ m}$, $x_4 = 1.2\text{ cm} = 1.2 \times 10^{-2}\text{ m}$

Position of nth bright fringe, $x_n = n\dfrac{D\lambda}{d}$

or $$x_4 = 4\frac{D\lambda}{d}$$

$\therefore$ $$\lambda = \frac{x_4\, d}{4D} = \frac{1.2 \times 10^{-2} \times 0.28 \times 10^{-3}}{4 \times 1.4} = 6 \times 10^{-7}\text{ m} = \mathbf{6000\ Å}$$

10.5. *In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ is k units. What is the intensity of light at a point where path difference is λ/3 ?*

Ans. Refer to the solution of Example 10(*ii*) on page 10.12.

10.6. *A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.*

(*a*) *Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.*

(*b*) *What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide ?*

The distance between the two slits is 2 mm and the distance between the plane of the slits and the screen is 120 cm.

Ans. Here $d = 2\text{ mm} = 2 \times 10^{-3}\text{ m}$, $D = 120\text{ cm} = 1.2\text{ m}$, $\lambda_1 = 650\text{ nm} = 650 \times 10^{-9}\text{ m}$, $\lambda_2 = 520\text{ nm} = 520 \times 10^{-9}\text{ m}$.

(*a*) The distance of the third bright fringe from the central maximum for wavelength 650 nm is

$$x_3 = \frac{3D\lambda_1}{d} = \frac{3 \times 1.2 \times 650 \times 10^{-9}}{2 \times 10^{-3}}\text{ m} = 1.17 \times 10^{-3}\text{ m} = \mathbf{1.17\ mm.}$$

(*b*) Suppose at any distance x from the central maximum, we have

$$x = n_1\beta_1 = n_2\beta_2$$

or $$n.\frac{D\lambda_1}{d} = n_2\frac{D\lambda_2}{d}$$

or $$n_1\lambda_1 = n_2\lambda_2$$

The bright fringes will coincide at the least distance x, if

$$n_2 = n_1 + 1$$

$\therefore$ $$n_1\lambda_1 = (n_1 + 1)\lambda_2$$

or $$n_1 \times 650 \times 10^{-9} = (n_1 + 1) \times 520 \times 10^{-9}$$

or $$n_1 = 4$$

Hence the required distance is

$$x = \frac{n_1 D\lambda_1}{d} = \frac{4 \times 1.2 \times 650 \times 10^{-9}}{2 \times 10^{-3}}\text{ m} = 1.56 \times 10^{-3}\text{ m} = \mathbf{1.56\ mm.}$$

10.7. *In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water ? Take refractive index of water to be 4/3.*

Ans. Since a fringe of width β is formed on the screen at distance D from the slits, so the angular fringe width would be

$$\theta = \frac{\beta}{D} = \frac{D\lambda/d}{D} = \frac{\lambda}{d}$$

or $$d = \frac{\lambda}{\theta}$$

If the wavelength in water be λ' and the angular fringe width be θ', then

$$d = \frac{\lambda'}{\theta'} \qquad \therefore \qquad \frac{\lambda}{\theta} = \frac{\lambda'}{\theta'}$$

or $$\theta' = \frac{\lambda'}{\lambda}.\theta = \frac{\lambda/\mu}{\lambda}.\theta = \frac{\theta}{\mu} = \frac{0.2°}{4/3} = \mathbf{0.15°}$$

10.8. *What is Brewster angle for air to glass transition ? (μ for glass is 1.5).*

Ans. By Brewster law,

$$\tan i_p = \mu = 1.5$$

$\therefore$ Brewster angle,

$$i_p = \tan^{-1}(1.5) = \mathbf{56.3°.}$$

10.9. *Light of wavelength 5000 Å falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light ? For what angle of incidence is the reflected ray normal to the incident ray ?*

Ans. The speed of light is fixed in a given medium and frequency of light does not change during its reflection from a surface. Consequently, the wavelength should also remain unchanged.

Speed of reflected light = Speed of light in air

or $$c = 3 \times 10^8\text{ ms}^{-1}$$

Wavelength of reflected light

= Wavelength of incident light

or $$\lambda = 5000\text{ Å} = 5 \times 10^{-7}\text{ m}$$

Frequency of reflected light,

$$\nu = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{5 \times 10^{-7}} = \mathbf{6 \times 10^{14}\ Hz}$$

By law of reflection, $\angle i = \angle r$

Given $\angle i + \angle r = 90°$

$\therefore \quad \angle i = \mathbf{45°}$.

10.10. *Estimate the distance for which ray optics is a good approximation for an aperture of 4 mm and wavelength 400 nm.*

Ans. Here $a = 4\text{ mm} = 4 \times 10^{-3}\text{ m}$,
$\lambda = 400\text{ nm} = 4 \times 10^{-7}\text{ m}$

$$D_F = \frac{a^2}{\lambda} = \frac{(4 \times 10^{-3})^2}{4 \times 10^{-7}} = \mathbf{40\ m.}$$

Thus ray optics is valid upto a distance of 40 m from the aperture.

10.11. *The 6563 Å H_α line emitted by hydrogen in a star is found to be red shifted by 15 Å. Estimate the speed with which the star is receding from the earth.*

Ans. Here $\lambda = 6563$ Å, $\Delta\lambda = 15$ Å

As $\quad \Delta\lambda = -\frac{v}{c}\lambda$

$$\therefore \quad v = -\frac{\Delta\lambda}{\lambda}\cdot c = -\frac{15}{6563} \times 3 \times 10^8 = \mathbf{-6.86 \times 10^5\ ms^{-1}}$$

The negative sign shows that the star is receding away from the earth.

10.12. *Explain how Newton's corpuscular theory predicts that the speed of light in a medium, say water, is greater than the speed of light in vacuum. Is the prediction confirmed by experimental determination of the speed of light in water ? If not, which alternative picture of light is consistent with experiment ?*

Ans. In Newton's corpuscular (particle) picture of refraction, particles of light incident from a rarer to a denser medium experience a force of attraction normal to the surface. This results in an increase in the normal component of velocity but the component along the surface remains unchanged.

Consider a ray of light going from a rarer medium (air) to a denser medium (water).

Let c = speed of light in vacuum (or air),
v = speed of light in water,
i = angle of incidence, and
r = angle of refraction.

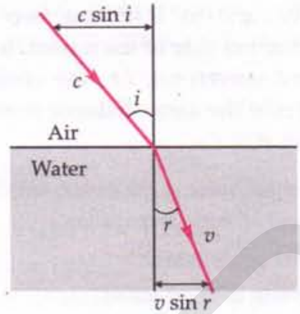


Fig. 10.77

Then according to Newton's corpuscular theory,

Component of velocity c
= Component of velocity v, along the surface of separation

$\therefore \quad c \sin i = v \sin r$

or $\quad \frac{v}{c} = \frac{\sin i}{\sin r} = {}^a\mu_\omega$

As ${}^a\mu_\omega > 1$, therefore, $v > c$.

Thus, Newton's corpuscular theory predicts that light should travel faster in water than in air. This prediction is *opposite to* the experimental result : $v < c$. The prediction of Huygens' wave theory is consistent with the experimental results.

10.13. *You have learnt in the text how Huygens' principle leads to the laws of reflection and refraction. Use the same principle to deduce directly that a point object placed in front of a plane mirror produces a virtual image whose distance from the mirror is equal to the object distance from the mirror.*

Ans. With the point object A as the centre, draw a circular arc WPW just touching the mirror at P. Then WPW represents the section of the spherical wave front that has just reached the mirror. According to Huygen's principle, every point on this wavefront is a source of secondary wavelets. During the time t in which the incident wavefront travels from W to W', the secondary wavelets from P must have spread over a distance $PP'' = WW'$. Similarly, other disturbance from other points of the mirror reaches the various points of the surface $W'P''W'$. In Fig. 10.78, $W'P'W'$ is the reflected wavefront at instant t, while $W'P'W'$ is the position of incident wavefront at the same instant t, in the absence of mirror.

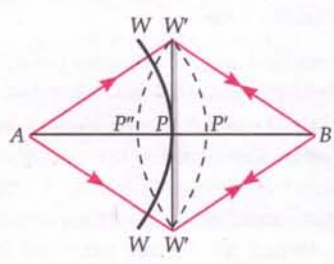


Fig. 10.78

As $W'P''W'$ and $W'P'W'$ are two symmetrically located arcs on either side of the mirror, hence the centre of the reflected wavefront, *i.e.*, the virtual image B of point object A is at the same distance from the mirror as the point object A.

10.14. *Let us list some of the factors which could possibly influence the speed of wave propagation :*

(i) nature of the source

(ii) direction of propagation

(iii) motion of the sources and/or observer

(iv) wavelength

(v) intensity of the wave

On which of these factors, if any, does

(a) the speed of light in vacuum

(b) the speed of light in a medium (say glass or water) depend ?

Ans. (*a*) **Dependence of speed of light in vacuum.** The speed of light in vacuum is a universal constant. It does not depend on the factors listed above. It is independent of the relative motion between the source and the observer. The *constantancy and absoluteness of the speed of light in vacuum* is one of the basic postulates of Einstein's special theory of relativity.

(*b*) **Dependence of speed of light in a medium :**

(*i*) The speed of light in a medium does not depend on the nature of the source. Wave speed is determined by the properties of the medium of propagation. This is also true of other waves, like sound waves, water waves etc.

(*ii*) The speed of light in a medium is independent of the direction of propagation for isotropic media.

(*iii*) The speed of light is independent of the motion of the source relative to the medium but it depends on the motion of the observer relative to the medium.

(*iv*) The speed of light in a medium depends on wavelength of light.

(*v*) The speed of light in a medium is independent of intensity. The same is not true for high intensity beams.

10.15. *For sound waves, the Doppler formula for frequency shift differs slightly between the two situations : (i) source at rest ; observer moving, and (ii) source moving ; observer at rest. The exact Doppler formulas for the case of light waves in vacuum are, however, strictly indentical for these situations. Explain why this should be so. Would you expect the formulas to be strictly identical for the two situations in case of light travelling in a medium ?*

Ans. Sound waves require a medium for propagation. Thus even though the situations (*i*) and (*ii*) may correspond to the same relative motion (between the source and the observer), they are not identical physically since the motion of the observer *relative to the medium* is different in the two situations. Therefore, we cannot expect Doppler formula for sound to be identical for (*i*) and (*ii*). For light waves in vacuum there is clearly nothing to distinguish between (*i*) and (*ii*). Here only the relative motion between the source and the observer counts and the relativistic formula is the same for (*i*) and (*ii*). For light propagation in a medium, once again like for sound waves, the two situations are not identical and we should expect the Doppler formula for this case to be different for the situations (*i*) and (*ii*).

10.16. *In double-slit experiment using light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is 0.1°. What is the spacing between the two slits ?* [CBSE D 15]

Ans. Since a fringe of width β is formed on the screen at distance D from the slits, therefore

$$\theta = \frac{\beta}{D} = \frac{D\lambda / d}{D} = \frac{\lambda}{d}$$

or $$d = \frac{\lambda}{\theta}$$

But $$\lambda = 600 \text{ nm} = 6 \times 10^{-7} \text{ m},$$

$$\theta = 0.1° = \frac{0.1}{180} \times \pi \text{ rad}$$

$$\therefore \quad d = \frac{6 \times 10^{-7} \times 180}{0.1 \times \pi} \text{ m}$$

$$= \mathbf{3.44 \times 10^{-4} \text{ m}.}$$

10.17. *Answer the following questions :*

(a) In a single-slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band ?

(b) In what way is diffraction from each slit related to the interference pattern in a double-slit experiment ? [CBSE D 13 ; OD 13C]

(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why. [CBSE D 09 ; OD 13C]

(d) Two students are separated by a 7 m partition wall in a room 10 m high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other even though they can converse easily ?

(e) Ray optics is based on the assumption that light travels in a straight line. Diffraction effects (observed when light propagates through small apertures / slits or around small obstacles) disprove this assumption. Yet the geometrical optics

assumption is so commonly used in understanding location and several other properties of images in optical instruments. What is the justification ?

Ans. (*a*) Width of central maximum $= \frac{2D\lambda}{a}$

On doubling the width '*a*' of the slit, the size of the central diffraction band is halved. But the amplitude of light intensity gets doubled. So the intensity becomes four times the initial intensity.

(*b*) The intensity of interference fringes in Young's double slit experiment is modified by the diffraction pattern of each slit.

(*c*) Waves from the distant source are diffracted by the edge of the circular obstacle and these diffracted waves interfere constructively at the centre of the obstacle's shadow producing a bright spot.

(*d*) For diffraction or bending by obstacles, apertures by a large angle the size of the latter should be comparable to the wavelength. If the size of the obstacle/aperture is much too large compared to λ, diffraction is by a small angle. The size of the partition wall is of the order of few metres. Wavelength of sound waves (0.3 m for 1 kHz frequency) is comparable to the size of the partition but wavelength of light is much small ($\approx 5\times10^{-7}$ m). So sound waves can bend around the partition while light moves cannot.

(*e*) In ordinary optical instruments, the sizes of apertures are much larger than the wavelength of light. So the diffraction effects are negligibly small. Hence the assumption that light travels in straight lines can be safely used in the optical instruments.

10.18. *Two towers on the top of two hills are 40 km apart. The line joining them passes 50 m above a hill half way between the towers. What is the longest wavelength of radiowaves which can be sent between the towers without appreciable diffraction effects ?*

Ans. Here size of Fresnel zone a_F at the middle hill must be less than 50 m.

Distance of either of the two hills from the middle hill is

$$D = \frac{40}{2}\text{ km} = 20{,}000\text{ m}$$

As size of Fresnel's zone, $a_F = \sqrt{\lambda D}$

$$\therefore \quad \sqrt{\lambda D} << 50$$

$$\text{or} \quad \lambda D << 2500$$

$$\text{or} \quad \lambda << \frac{2500}{D}$$

$$= \frac{2500}{20{,}000} = 0.125\text{ m} = \mathbf{12.5\ cm.}$$

Thus wavelengths longer than 12.5 cm will undergo appreciable diffraction effects.

10.19. *A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit.* **[CBSE OD 13]**

Ans. Here $\lambda = 500\text{ nm} = 5\times10^{-7}\text{ m}$, $D = 1\text{ m}$, $x_1 = 2.5\text{ mm} = 2.5\times10^{-3}\text{ m}$

As $x_1 = \frac{D\lambda}{a}$

$$\therefore \quad a = \frac{D\lambda}{x_1} = \frac{1\times5\times10^{-7}}{2.5\times10^{-3}} = 2\times10^{-4}\text{ m} = \mathbf{0.2\ mm.}$$

10.20. *Answer the following questions :*

(a) When a low-flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV screen. Suggest a possible explanation.

(b) As you have learnt in the text, the principle of linear superposition of wave displacements is basic to understanding intensity distributions in diffraction and interference patterns. What is the justification of this principle ?

Ans. (*a*) The low flying aircraft reflects the TV signals. Due to interference between the direct signal received by the antenna and the (weak) reflected signal, we sometimes observe slight shaking of the picture on the TV screen.

(*e*) Superposition principle follows from the linear character of the (differential) equation governing wave motion. If y_1 and y_2 are solutions of the wave equation, so is any linear combination of y_1 and y_2. When the amplitudes are large (*e.g.*, high intensity laser beams) and non-linear effects become important, the situation is far more complicated.

10.21. *In deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles of $\frac{n\lambda}{a}$. Justify this by suitably dividing the slit to bring out the cancellation.*

Ans. Divide the single slit into *n* smaller slits of width $a' = \frac{a}{n}$.

Then angle,

$$\theta = \frac{n\lambda}{a} = \frac{\lambda}{a'}.$$

Each of the smaller slits sends zero intensity in the direction θ. The combination gives zero intensity as well.

Text Based Exercises

TYPE A : VERY SHORT ANSWER QUESTIONS (1 mark each)

1. Name the scientist who first proposed the wave theory of light.
2. Which three phenomena establish the wave nature of light ? [Punjab 2000]
3. Define the term 'wavefront'. [ISCE 98 ; CBSE OD 03, 04C ; F 09, 12]
4. Define a ray of light. [Punjab 97, 98]
5. How is the direction of a ray related to a wavefront ? [Haryana 95]
6. What is the phase difference between two points on a wavefront ? [Himachal 04]
7. What is the geometrical shape of the wavefront of light diverging from a point source. [ISCE 01 ; CBSE D 2000, 05]
8. What is the shape of wavefront emitted by a light source in the form of a narrow slit ? [Haryana 02]
9. A small piece of stone is dropped into a pond of still water. What is the shape of wavefront ?
10. A spherical wavefront is increasing in size. Whether the rays constitute a convergent or a divergent beam ?
11. Rays of light are converging to a point. Is the radius of the spherical wavefront increasing or decreasing in size ?
12. What is the shape of the wavefront of light emerging out of a convex lens when a point source is placed at its focus ?
13. What will be the shape of wavefront of light coming from a point source placed at infinity ?
14. Which principle is used to find the position of a wavefront after some time ?
15. State Huygens' principle. [ISÇE 2000, 03]
16. Sketch the wavefronts corresponding to converging rays. [CBSE D 98]
17. Sketch the wavefronts corresponding to diverging rays. [CBSE D 98]
18. Sketch the refracted wavefront emerging from a convex lens if a plane wavefront is incident normally on it. [CBSE D 09 ; OD 14C]
19. Light of wavelength 5000 Å propagating in air gets partly reflected from the surface of water. How will the wavelengths and frequencies of the reflected and refracted light be affected ? [CBSE D 15]
20. What is interference of light ? [Haryana 02, 04]
21. Draw the shape of the wavefront coming out of a concave mirror when a plane wave is incident on it. [CBSE OD 14C]
22. State the principle of superposition of waves.
23. What is the main condition to produce interference of light ? [ISC 95]
24. Define the term 'coherent sources' which are required to produce interference pattern in Young's double slit experiment. [CBSE C]
25. State the conditions which must be satisfied for two light sources to be coherent. [Punjab 95 ; CBSE OD 15]
26. Give a relation between path difference and wavelength for constructive interference between two waves. [Himachal 98 ; CBSE D 92 ; Haryana 02]
27. State the path difference between two waves for destructive interference. [CBSE D 92 ; Haryana 02]
28. What is the effect on the interference fringes in a Young's double slit experiment if the screen is moved away from the plane of the slits ? [CBSE OD, 01C, 10]
29. What will be the effect on the interference fringes in Young's double slit experiment, if the monochromatic source is replaced by another monochromatic source of shorter wavelength ? [CBSE F 1994]
30. How can the fringe width be increased in Young's double slit experiment ?
31. What is the shape of interference fringes in Young's double slit experiment ?
32. Is the central fringe bright or dark in Young's double slit experiment ?
33. How much is the distance in terms of fringe width β between central bright and fourth dark fringe ?
34. Can we observe interference maxima on the screen if the two slits are separated by less than a wavelength of light used ?
35. What is the ratio of the fringe width for bright and dark fringes in Young's double slit experiment ?
36. Does interference phenomenon violate the law of conservation of energy ?
37. Does interference phenomenon reveal the nature of light waves ?

38. What is effect on interference fringes in Young's double slit experiment if one slit is covered ?
39. Obtain the ratio of the interference fringe widths β_1 and β_2 obtained with monochromatic red light of $\lambda_1 = 660$ nm and ultraviolet light of $\lambda_2 = 330$ nm. [ISCE 03]
40. Two slits in Young's double slit experiment have widths in the ratio 81 : 1. What is the ratio of the amplitudes of light waves from them ?
41. What is the ratio of the slit widths when amplitudes of light waves from them have a ratio $\sqrt{2}:1$?
42. Two waves of amplitudes 3 m and 2 m reach a point in the same phase. What is the resultant amplitude ?
43. Two waves of amplitudes 3 mm and 5 mm reach a point in opposite phases. What is the resultant amplitude ?
44. The phase difference between two waves reaching a point is $\pi/2$. What is the resultant amplitude if the individual amplitudes are 3 mm and 4 mm ?
45. In Young's double slit experiment, two disturbances arriving at a point P have a phase difference of $\pi/3$. What is the intensity at this point expressed as a fraction of maximum intensity I_0 ?
46. Two coherent sources emit waves of amplitudes a and $2a$. They meet at a point P equidistant from the two sources. If the intensity of the first is I, what is the resultant intensity at point P ?
47. Thin films, such as a layer of oil on water, show beautiful colours when illuminated by white light. Name the phenomenon involved.
48. What is diffraction of light ? [Haryana 02 ; Himachal 03]
49. What should be the approximate slit size to observe diffraction with it ? [Himachal 1994]
50. What is the condition for first minimum in case of diffraction due to single slit ? [Haryana 93]
51. What is Fresnel's distance ?
52. A diffraction grating has 5000 lines per cm. What is its grating element ? [ISCE 95]
53. Define resolving power of an optical instrument. How does it depend on wavelength ? [ISCE 94]
54. What is meant by the term angular resolution of a telescope ? [CBSE OD 03C]
55. How are resolving power and limit of resolution of an optical instrument related ? [Himachal 95]
56. Define resolving power of a microscope. [CBSE OD 92]
57. Define resolving power of a telescope. [CBSE D 92]
58. How can we increase the resolving power of a microscope ?
59. Write the factors by which the resolving power of a telescope can be increased. [CBSE OD 15C]
60. Name one such phenomenon that is shown by light waves but not by sound waves.
61. Which phenomenon leads us to conclude that light has transverse wave nature ?
62. What do you mean by polarisation of light ? [Punjab 02 ; CBSE OD 02]
63. What is plane polarised light ? [ISCE 97 ; CBSE OD 2000C]
64. Define circularly polarised light. [CBSE D 92]
65. Define elliptically polarised light. [CBSE D 92]
66. Is light from a sodium lamp plane polarised ?
67. Can electromagnetic waves be polarised ?
68. Can you recognise with the naked eye whether the given light is polarised or not ?
69. Is the blue light from the sky polarised or not ?
70. Which among X-rays, sound waves and radiowaves can be polarised ?
71. A beam of light is incident on a polaroid. On rotating the polaroid through 360° about the incident ray as the axis, the intensity of transmitted light varies between a maximum and a non-zero minimum. What can you say about the state of polarisation of the incident light ?
72. State the law of Malus.
73. Define the polarisation angle for polarisation by reflection. [Punjab 02]
74. What is meant by polarising angle ? [ISCE 95]

Or

Define Brewster angle.
75. State Brewster law for polarisation of light. [ISCE 2000 ; Punjab 98C ; Haryana 2000, 04]
76. A ray of light is incident on a medium at the polarizing angle. What is the angle between the reflected and refracted rays ? [ISCE 97]
77. What is the polarising angle of a medium in which the angle of refraction is 33° ? [CBSE D 2000]
78. What is the value of refractive index of a medium of polarising angle 60° ? [CBSE OD 2000]
79. What is a polaroid ? [ISCE 95 ; Pb 2000, 02 ; CBSE OD 01C]
80. Give two examples of commonly used devices which make use of polaroids.

81. A polariser and analyser are so oriented that no light is transmitted. What is the angle between the axes of polariser and analyser ?
82. A polariser and an analyser are so oriented that intensity of transmitted light is maximum. If the analyser is rotated through 60°, what fraction of the maximum light is transmitted ?
83. What percentage of the incident light is transmitted if the angle between polariser and analyser is 30°?
84. When light is polarised by reflection, what is the direction of vibration of the electric field vector of the polarised light ?
85. Why do we see two images of an object when looked through a calcite crystal ?
86. Draw a sketch showing the incident, reflected and transmitted rays when light is incident at the polarising angle on a glass slab. [ISCE 94]
87. How would the angular separation of interference fringes in Young's double slit experiment change when the distance of separation between the slits and the screen is doubled ? [CBSE OD 09]
88. How does the angular separation of interference fringes change, in Young's experiment, if the distance between the slits is increased ? [CBSE D 08]
89. How does the fringe width of interference fringes change, when the whole apparatus of Young's experiment is kept in a liquid of refractive index 1.3 ? [CBSE D 08]
90. A parallel beam of monochromatic light falls normally on a single slit. How does the angular width of principal maximum in the diffraction pattern depend on the width of the slit. [CBSE OD 08 C]
91. State one feature by which the phenomenon of interference can be distinguished from that of diffraction. [CBSE OD 08 C]
92. What is the angle between the plane of the polariser and that of an analyser, in order that the intensity of the transmitted light reduces to half ? [CBSE F 08]
93. Differentiate between a ray and a wavefront. [CBSE D 09]
94. What type of wavefront will emerge from a (*i*) point source, and (*ii*) distant light source ? [CBSE D 09]
95. Sketch the shape of wavefront emerging from a point source of light and also mark the rays. [CBSE F 09]
96. Why are coherent sources required to create interference of light ? [CBSE F 09]
97. Unpolarised light of intensity I is passed through a polaroid. What is the intensity of the light transmitted by the polaroid ? [CBSE F 09]
98. Unpolarised light is incident on a plane surface of glass of refractive index μ at angle i. If the reflected light gets totally polarizeed, write the relation between the angle i and refractive index μ. [CBSE D 09]

Answers

1. Christian Huygens.
2. Phenomena like interference, diffraction and polarization confirm the wave nature of light.
3. The locus of all such particles of the medium which are vibrating in the same phase is called a wavefront.
4. A line drawn perpendicular to a wavefront in the direction of propagation of light wave is called a ray of light.
5. The direction of a ray is normal to a wavefront.
6. Zero.
7. Spherical, with point source at the centre of the sphere.
8. Cylindrical shape.
9. Circular.
10. Divergent beam.
11. The radius of the wavefront is decreasing.
12. The emerging light has plane wavefronts.
13. Plane.
14. Huygens' principle of secondary wavelets.
15. According to Huygens' principle,
    (*i*) Each point on a wavefront becomes a fresh source of secondary wavelets, which spread out with the speed of light in that medium.
    (*ii*) The new wavefront at any later time is given by the forward envelope of the secondary wavelets at that time.
16. See Fig. 10.2(*b*) on page 10.2.
17. See Fig. 10.2(*c*) on page 10.2.
18. See Fig. 10.8 on page 10.5.
19. *For reflected light*. Wavelength remains the same. Also frequency remains the same.
    For refracted light. Wavelength decreases. But frequency remains the same.
20. When two waves of same frequency and having zero or constant phase difference travelling in the same direction superpose each other, the intensity in the region of superposition gets redistributed, becoming maximum at some points and minimum at others. This phenomenon is called interference of waves.

21.

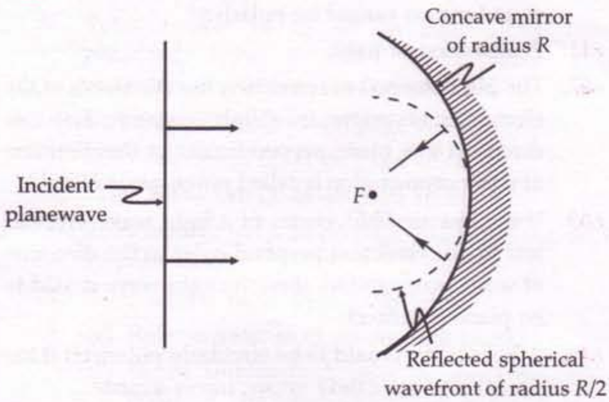


Fig. 10.79

22. Refer answer to Q. 8 on page 10.8.
23. The sources of light must be coherent.
24. Two sources of light are said to be coherent if they continuously emit light waves of the same frequency (or wavelength) with a zero or constant phase difference between them.
25. Two light sources will be coherent if
    (*i*) the frequency of the two light sources is same, and
    (*ii*) the phase difference between them remains constant.
26. For constructive interference, the path difference between two waves must be integral multiple of λ *i.e.*, $p = n\lambda$, $n = 0, 1, 2, 3....$
27. For destructive interference, the path difference between two waves must be odd multiple of $\lambda/2$ *i.e.*,

$$p = (2n-1)\frac{\lambda}{2}, \quad n = 1, 2, 3....$$

28. As fringe width $\beta = \frac{D\lambda}{d}$, so as the screen is moved away from the slits (D increases), fringe width increases.
29. Fringe width decreases because

$$\beta = \frac{D\lambda}{d} \text{ i.e., } \beta \propto \lambda.$$

30. Fringe width, $\beta = \frac{\lambda D}{d}$

Clearly fringe width can be increased by

(*i*) increasing the distance D between the coherent sources and the screen.

(*ii*) decreasing the distance d between the two coherent sources,

(*iii*) using light of higher wavelength λ.

31. Nearly straight line fringes, parallel to the slits.
32. The central fringe is bright in Young's double slit experiment.
33. Distance between central bright fringe and fourth dark fringe

$$= \frac{7}{2}\beta, \text{ where } \beta \text{ is fringe width.}$$

34. No, fringe width will become very large.
35. 1 : 1, because both bright and dark fringes have same fringe width.
36. No, it simply redistributes energy in the medium.
37. No, because interference occurs both in transverse and longitudinal waves.
38. Diffraction pattern replaces the interference pattern.
39. $\frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2} = \frac{660}{330} = \mathbf{2:1}$.
40. Width ratio, $\frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{81}{1}$

$\therefore$ Amplitude ratio,

$$\frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{81}{1}} = \mathbf{9:1}$$

41. Width ratio,

$$\frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \left(\frac{\sqrt{2}}{1}\right)^2 = \mathbf{2:1}$$

42. Resultant amplitude = 3 + 2 = **5 m**.
43. Resultant amplitude = 5 − 3 = **2 mm**.
44. Resultant amplitude,

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\frac{\pi}{2}} = \sqrt{3^2 + 4^2} = \mathbf{5\ mm}.$$

45. The resultant intensity at any point of an interference pattern is given by

$$I = I_0 \cos^2\frac{\phi}{2}$$

where I_0 is the maximum value of I and ϕ is the phase difference between two waves. Given $\phi = \pi/3$, therefore,

$$I = I_0 \cos^2\frac{\pi}{6} = I_0\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}I_0.$$

46. Resultant amplitude at point $P = a + 2a = 3a$.

As intensity $\propto$ (amplitude)^2, therefore

Resultant intensity at point $P = 9I$.

47. Interference of light.
48. The phenomenon of bending of light around the corners of small obstacles or apertures and its spreading into the regions of geometrical shadow is called diffraction of light.

49. The size of the slit should be of the order of wavelength of light used.

50. If a is slit width, then condition for first minimum is $a \sin \theta = \lambda$.

51. Fresnel's distance is the distance at which the diffraction spread of a beam becomes equal to the size of the aperture.

52. Grating element $= \frac{1}{5000} = \mathbf{2 \times 10^{-4}\ cm}$.

53. The resolving power of an optical instrument is reciprocal of the smallest linear or angular separation between two point objects, whose images can be just resolved by the instrument.

The resolving power of an optical instrument is inversely proportional to the wavelength of light used.

54. The angular resolution of a telescope is defined as the reciprocal of the smallest angular separation between two point objects whose images can be just resolved by it.

55. Resolving power of an optical instrument

$$= \frac{1}{\text{Limit of resolution}}.$$

56. Resolving power of a microscope is defined as the reciprocal of the smallest distance between two point objects which can just be resolved by it.

$$\text{Resolving power} = \frac{1}{d} = \frac{2\mu \sin\theta}{\lambda}$$

Here θ is half the angle of cone of light from point object on to the objective lens.

57. Resolving power of a telescope is defined as the reciprocal of the smallest angular separation between two point objects whose images can be just resolved by it.

Resolving power of a telescope

$$= \frac{1}{d\theta} = \frac{D}{1.22\,\lambda}$$

where D = diameter of the telescope objective

and $d\theta$ = angle subtended by the two distant objects at the objective.

58. The resolving power of a microscope can be increased.

(*i*) by using light of smaller wavelength λ.

(*ii*) by increasing refractive index of the medium between the objective and the object.

59. The resolving power of telescope can be increased by

(*i*) increasing the diameter of its objective.

(*ii*) using light of shorter wavelength.

60. Polarisation. Light waves can be polarised while sound waves cannot be polarised.

61. Polarisation of light.

62. The phenomenon of restricting the vibrations of the electric field vector of a light wave to just one direction in a plane perpendicular to the direction of wave propagation is called polarisation of light.

63. If the electric field vector of a light wave vibrates just in one direction perpendicular to the direction of wave propagation, then the light wave is said to be plane polarised.

64. A light wave is said to be circularly polarised if the tip of its electric field vector traces a circle.

65. A light wave is said to be elliptically polarised if the tip of its electric field vector traces an ellipse.

66. No.

67. Yes, because they are transverse in nature.

68. No. This can be done only with the help of a polaroid.

69. It is polarised.

70. X-rays and radio waves can be polarised because they are transverse waves.

71. The incident light is partially polarised.

72. According to the law of Malus, when a plane polarised light is passed through an analyser, the intensity I of the transmitted light varies directly as the square of the cosine of the angle θ between the transmission directions of polariser and analyser. Mathematically

$$I = I_0 \cos^2 \theta$$

Here I_0 is the maximum intensity of the transmitted light.

73, 74. The angle of incidence, at which when unpolarised light is incident on a transparent refracting medium, the reflected light becomes completely plane polarising is called polaring angle or Brewster angle.

75. Brewster law states that the tangent of the polarising angle of incidence for a transparent refracting medium is equal to the refractive index of that medium. Mathematically,

$$\tan i_p = \mu$$

76. 90°.

77. $i_p = 90° - r_p = 90° - 33° = 57°$.

78. $\mu = \tan i_p = \tan 60° = \sqrt{3}$.

79. A polaroid is a thin commercial sheet which makes use of the property of selective absorption to produce an intense beam of plane polarised light.

80. (*i*) Sun glasses
(*ii*) Liquid crystal displays (LCDs).

81. 90°

82. From Malus law,

$$I = I_0 \cos^2 60° = \frac{I_0}{4}$$

$$\therefore \quad \frac{I}{I_0} = \frac{1}{4}.$$

83. From Malus law,

$$I = I_0 \cos^2 30° = \frac{3}{4} I_0$$

$\therefore$ Percentage of incident light transmitted

$$= \frac{I}{I_0} \times 100 = \frac{3}{4} \times 100 = 75.$$

84. The electric field vector vibrates perpendicular to the plane of incidence.

85. This is due to double refraction of light by the calcite crystal.

86. See Fig. 10.44 on page 10.48.

87. Angular separation, $\theta = \frac{\lambda}{d}$

When the distance D of separation between the slits and the screen is doubled, the angular separation θ remains unchanged.

88. Angular separation between the interference fringes decreases, because $\theta \propto \frac{1}{d}$.

89. Wavelength of light in the liquid $(\lambda' = \lambda / 1.3)$ decreases. Hence fringe width $(\beta \propto \lambda)$ also decreases.

90. Angular width of principal maximum, $2\theta = \frac{2\lambda}{a}$.

Clearly, it is inversely proportional to the width of the slit.

91. In an interference pattern, all bright fringes are of same intensity. In a diffraction pattern, the intensity of bright fringes decreases with the increase in distance from the central bright fringe.

92. From Malus law, $I = I_0 \cos^2 \theta$

$$\therefore \quad \frac{I_0}{2} = I_0 \cos^2 \theta$$

or $\quad \cos^2 \theta = \frac{1}{2} \quad$ or $\quad \cos \theta = \frac{1}{\sqrt{2}}$

Hence $\quad \theta = 45°$.

93. A ray represents the path along which a wave travels. A wavefront is the locus of all points in the wave vibrating in the same phase.

94. (*i*) Spherical wavefront. (*ii*) Plane wavefront.

95. See Fig. 10.1(*a*).

96. In order to obtain an observable sustained interference pattern, two coherent sources of light are necessary.

97. $I/2$, as half the incident unpolarised light is cut off by the polaroid.

98. $\mu = \tan i$.

TYPE B : SHORT ANSWER TYPE QUESTIONS (2 or 3 marks each)

1. Define the term wavefront ? What are the assumptions on which Huygens' principle is based ? Describe Huygens' geometrical construction for propagation of wavefronts in a medium. **[Himachal 04]**

2. State Huygens' principle. Use it to show that a plane wavefront advances as a plane wavefront in a homogeneous medium. Is a backward wavefront possible ? Give reason in support of your answer.

3. State Huygens' postulates of wave theory. Sketch the wavefront emerging from a (*i*) point source of light and (*ii*) linear source of light like a slit. **[CBSE D 2000, 09C]**

4. State the postulates of Huygens' wave theory. Sketch the wave front that corresponds to a beam of light (*i*) coming from a very far away source, and (*ii*) diverging radially from a point source. **[CBSE OD 01, 03C]**

5. What is wavefront ? Distinguish between a plane wavefront and a spherical wavefront. Explain with the help of a diagram, the refraction of a plane wavefront at a plane surface using Huygen's construction. **[CBSE Sample Paper 08]**

6. What is a wave front ? What is the geometrical shape of a wave front of light emerging out of a convex lens, when point source is placed at its focus ? Using Huygens' principle show that, for a parallel beam incident on a reflecting surface, the angle of reflection is equal to the angle of incidence. **[CBSE D 2000C ; OD 03]**

7. Deduce the laws of reflection on the basis of Huygens' wave theory. **[Himachal 02, 03, 04 ; Punjab 01]**

8. A plane wavefront propagating in a medium of refractive index 'μ_1' is incident on a plane surface making the angle of incidence 'i' as shown in Fig. 10.80. It enters into a medium of refractive

index 'μ_2' ($\mu_2 > \mu_1$). Use Huygens' construction of secondary wavelets to trace the propagation of the refracted wavefront. Hence verify Snell's law of refraction. [CBSE F 15]

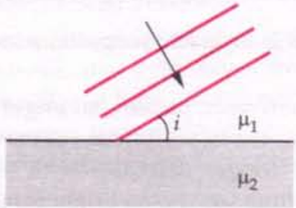


Fig. 10.80

9. State Huygens' principle and use it to construct refracted wavefront for refraction of a plane wave front at a plane refracting surface. Hence derive Snell's Law. [CBSE Sample Paper 03 ; D 2000C, 02C]
10. How is a wavefront defined ? Using Huygen's construction draw a figure showing the propagation of a plane wave refracting at a plane surface separating two media. Hence verify Snell's law of refraction. [CBSE D 08]
11. Show that a plane wavefront incident on a thin prism is tilted towards its base after refraction through it.
12. Draw the diagrams to show the behaviour of plane wavefronts as they pass through
    (*a*) a thin prism and
    (*b*) a thin convex lens. [CBSE 08 C]
13. (*a*) Use Huygens' geometrical construction to show the behaviour of a plane wavefront
    (*i*) passing through a biconvex lens ;
    (*ii*) reflecting by a concave mirror.
    (*b*) When monochromatic light is incident on a surface separating two media, why does refracted light have the same frequency as that of the incident light ? [CBSE F 12]
14. What is meant by the term 'interference of light' ? Write any two conditions necessary for obtaining well-defined and sustained interference pattern of light. [CBSE Sample Paper 1997]
15. What is interference of light ? Give one example from daily life. State the necessary conditions for sustained interference. [Punjab 04]
16. What are coherent sources of light ? Deduce an expression for the intensity at any point on the screen in Young's double slit experiment. [CBSE D 01]
17. Derive an expression for the intensity at any point on the observation screen in Young's double slit experiment. Hence write the conditions for constructive and destructive interference.
18. Why cannot two independent monochromatic sources produce sustained interference pattern ? Explain. [CBSE F 15]

    Or

    Can two different bulbs, similar in all respects, act as coherent sources ? Give reasons for your answer.
19. Describe, with the aid of a labelled diagram, how the wavelength of monochromatic light may be found by Young's double slit experiment. [CBSE D 93C]
20. Describe Young's double slit experiment to produce interference pattern due to a monochromatic source of light. Deduce the expression for the fringe width. [CBSE D 11 ; OD 14]
21. Does the appearance of bright and dark fringes in the interference pattern violate, in any way, conservation of energy ? Explain. [CBSE OD 15C]
22. Two sources of intensity I_1 and I_2 undergo interference in Young's double slit experiment.

    Show that $\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$,

    where a_1 and a_2 are the amplitudes of disturbance of two sources S_1 and S_2. [CBSE D 01C]
23. State two conditions for sustained interference of light. Draw the variation of intensity with position, in the interference pattern of Young's double slit experiment. [CBSE F 99 ; D 02]
24. Consider a monochromatic ray incident on a film of uniform thickness t and refractive index μ. Derive the condition for a dark fringe, when viewed from the same side of the film. [ISCE 97]
25. Explain colour formation in thin films.
26. Draw a labelled diagram to show how interference can take place in thin transparent films by reflected light. [ISCE 97]
27. Prove that the fringe width of both the bright and dark fringes in interference is same in Young's double slit experiment. [Punjab 99C ; Himachal 2000]
28. Obtain the relation, $a \sin \theta = \lambda$ for the first minimum of the diffraction pattern of a single slit of width a using light of wavelength λ. [ISCE 01]
29. How is Huygen's principle used to obtain the diffraction pattern due to a single slit ? Show the plot of variation of intensity with angle and state the reason for the reduction in intensity of secondary maxima compared to central maximum. [CBSE F 08]
30. A parallel beam of monochromatic light falls normally on a narrow slit of width 'a' to produce a

diffraction pattern on the screen placed parallel to the plane of the slit. Use Huygens' principle to explain that

(i) the central bright maxima is twice as wide as the other maxima.

(ii) the intensity falls as we move to successive maxima away from the centre on either side. [CBSE D 14C]

31. Explain Fraunhoffer diffraction at a single slit and derive the relation for linear width of central maximum. [Punjab 98, 99]

32. (a) Use Huygens' principle to explain the formation of diffraction pattern due to a single slit illuminated by a monochromatic source of light.

(b) When the width of the slit is made double the original width, how would this affect the size and intensity of the central diffraction band ? [CBSE D 12]

33. Give differences between interference and diffraction of light. [CBSE D 09 ; OD 13C]

Or

Write the important characteristic features which distinguish the interference pattern from those seen in a coherently illuminated single slit. [CBSE D 15C, OD 15]

34. Define resolving power of a compound microscope. On what factors does it depend ? [Punjab 2000, 01, 03]

35. Define resolving power of a telescope. On what factors does it depend ? [Punjab 01, 03]

36. Define resolving power of a telescope. How does diffraction limit its resolving power. [CBSE OD 99C]

37. Differentiate between polarised and unpolarised lights. How are these represented ? [Punjab 01]

38. With the help of a diagram, explain plane of vibration and plane of polarisation.

39. (a) Describe briefly, with the help of a suitable diagram, how the transverse nature of light can be demonstrated by the phenomenon of polarization.

(b) When the unpolarised light passes from air to a transparent medium, under what condition does the reflected light get polarized ? [CBSE D 11]

40. What is meant by plane polarised light ? What type of waves show the property of polarisation ? Describe a method for producing a beam of plane polarised light. [CBSE OD 2000C]

41. What is an unpolarized light ? Explain with the help of suitable ray diagram how an unpolarized light can be polarized by reflection from a transparent medium. Write the expression for Brewster angle in terms of the refractive index of denser medium. [CBSE D 10, OD 13C]

42. What is a polaroid ? How is it used to obtain plane polarized light from ordinary (unpolarized) light ? [ISCE 01]

43. Give a method for producing a beam of plane polarized light. Show how you will detect the presence of plane polarized light. Give one practical use of polarized light. [ISCE 97]

44. The polarisation of a beam of light by reflection, is best achieved when the reflected and refracted rays are at right angles to each other. Show that the polarising angle of incidence is then given by $i_p = \tan^{-1} \mu$. [CBSE Sample Paper 03]

45. Show, giving a suitable diagram, how unpolarized light can be polarised by reflection. [CBSE OD 14C]

46. How can one distinguish between an unpolarized light beam and a linearly polarized light beam using a polaroid ? [CBSE OD 96]

47. State Malus law. Unpolarised light of intensity I_0 passes through two polarides P_1 and P_2 such that the pass axis of P_2 makes an angle θ with the pass axis of P_1. Plot a graph showing the variation of intensity of light transmitted through P_2 as the angle θ varies from zero to 180°. [CBSE OD 14C]

48. Show, with the help of a diagram, how unpolarised sunlight gets polarised due to scattering. [CBSE OD 14C]

Or

Explain why on viewing clear blue portion of the sky through a polaroid the intensity of transmitted light varies as polaroid is rotated ?

49. (a) Describe briefly using a diagram how sunlight is polarised.

(b) Unpolarised light is incident on a polaroid. How would the intensity of transmitted light change when the polaroid is rotated ? [CBSE OD 13]

50. Define polarising angle. Derive the relation connecting polarising angle and the refractive index of a medium. [CBSE OD 01]

51. Distinguish between unpolarised and plane polarised light. An unpolarised light is incident on the boundary between two transparent media. State the condition when the reflected wave is totally plane polarised. Find out the expression for the angle of incidence in this case. [CBSE OD 08]

52. Define the term 'linearly polarised light'.
When does the intensity of transmitted light become maximum, when a polaroid sheet is rotated between two crossed polaroids ? [CBSE OD 09]

53. What does a polaroid consist of ? Show, using a simple polaroid, that light waves are transverse in nature. Intensity of light coming out of a polaroid does not change irrespective of the orientation of the pass axis of the polaroid. Explain why. [CBSE OD 15]

54. State clearly how an unpolarised light gets linearly polarised when passed through a polaroid.
(*i*) Unpolarised light of intensity I_0 is incident on a polaroid P_1 which is kept near another polaroid P_2 whose pass axis is parallel to that of P_1. How will the intensities of light, I_1 and I_2, transmitted by the polaroids P_1 and P_2 respectively, change on rotating P_1 without disturbing P_2 ?
(*ii*) Write the relation between the intensities I_1 and I_2. [CBSE OD 15]

55. In a single slit diffraction experiment, a monochromatic source of light of wavelength λ illuminates a narrow slit of width *a*. Show, giving appropriate reasoning, that the half angular width of the central maximum in the observed pattern is (nearly) equal to λ / a. [CBSE D 09C]

Answers

1. Refer answer to Q. 3 on page 10.2.
2. Refer answer to Q. 3 on page 10.2.
3. Refer answer to Q. 3 on page 10.2. See Fig. 10.1(*a*) and (*b*) on page 10.2.
4. Refer answer to Q. 3 on page 10.2. See Fig. 10.2(*a*) and (*b*).
5. Refer answer to Q. 2. on page 10.1 and Q. 5 on page 10.4.
6. Plane wavefront. Refer answer to Q. 4 on page 10.3.
7. Refer answer to Q. 4 on page 10.3.
8. Refer answer to Q. 5 on page 10.4.
9. Refer answer to Q. 5 on page 10.4.
10. Refer answer to Q. 5 on page 10.4.
11. Refer answer to Q. 7 on page 10.5.
12. See Fig. 10.7 and Fig. 10.8 on page 10.5.
13. (*a*) (*i*) See Fig. 10.8 on page 10.5.
(*ii*) See Fig. 10.9 on page 10.5.
(*b*) Refer to the solution of Problem 1(*a*) on page 10.61.
14. Refer to points 6 and 12 of Glimpses on page 10.101 and 10.102.
15. Refer to points 6 and 12 of Glimpses on page 10.101 and 10.102. A thin film of oil spread over water shows interference of light due to the interference between light waves reflected by the lower and upper surfaces of the thin film.
16. Refer answer to Q. 10 and Q. 11 on page 10.10.
17. Refer answer to Q. 10 on page 10.10.
18. Refer answer to Q. 11 on page 10.10.
19. Refer answer to Q. 13 on page 10.13.
20. Refer answer to Q. 13 on page 10.13.
21. Refer answer to Q. 16 on page 10.21.
22. Refer answer to Q. 17 on page 10.21.
23. Refer answer to Q. 14 on page 10.20. See Fig. 10.15 on page 10.21.
24. Refer answer to Q. 11 on page 10.10.
25. Refer answer to Q. 19 on page 10.23.
26. Refer answer to Q. 19 on page 10.23.
27. See Fig. 10.16 on page 10.24.
28. Refer answer to Q. 13 on page 10.13.
29. Refer answer to Q. 24 on page 10.30.
30. (*i*) Refer answer to Q. 25 on page 10.32.
(*ii*) Refer answer to Q. 24 on page 10.31.
31. Refer answer to Q. 25 on page 10.32.
32. (*a*) Refer answer to Q. 24 on page 10.30.
(*b*) Refer to the solution of Problem 25 on page 10.66.
33. Refer answer to Q. 28 on page 10.38.
34. Refer answer to Q. 31 on page 10.39.
35. Refer answer to Q. 32 on page 10.40.
36. Refer answer to Q. 32 on page 10.40 and Q. 29 on page 10.39.
37. Refer answer to Q. 35 on page 10.43.
38. Refer answer to Q. 39 on page 10.47.
39. (*a*) Refer answer to Q. 37 on page 10.44.
(*b*) When the unpolarised light falls on the transparent surface at an angle of incidence equal to polarising or Brewster angle (*i*), the reflected light is completely polarised.
40. Only transverse waves show the property of polarisation. Refer answer to Q. 44 on page 10.51.
41. Refer answer to Q. 42 on page 10.48.

42. Refer answer to Q. 46 on page 10.52.

43. Refer answer to Q. 48 on page 10.53. Plane polarised light is used to produce three dimensional effects in motion pictures.

44. Refer answer to Q. 42 on page 10.48.

45. Refer answer to Q. 42 on page 10.48.

46. Refer answer to Q. 48 on page 10.53.

47. Refer answer to Q. 38 on page 10.44. See Fig. 10.39 on page 10.45.

48. Refer answer to Q. 43 on page 10.50.

49. (*a*) Refer answer to Q. 43 on page 10.50.

(*b*) When unpolarised light is seen through a polaroid, the intensity of light is cut down to half due to polarisation of light. When the polaroid is rotated, the intensity of light does not change.

50. Refer answer to Q. 42 on page 10.48.

51. Refer answer to Q. 35 on page 10.45 and Q. 42 on page 10.48.

52. If the electric field vector of a light wave vibrates just in one direction perpendicular to the direction of wave propagation, then it is said to be liearly polarised.

The intensity of transmitted light becomes maximum when the polaroid sheet is placed at an angle of 45° with the pass axis (of the polariser) between the two crossed polaroids.

53. A polaroid consists of long chain molecules aligned in a particular direction.

For transverse wave nature of light, refer answer to Q. 37 on page 10.44.

When unpolarised light falls on a polaroid, only the vibrations parallel to the transmission plane get transmitted and perpendicular vibrations are absorbed. The intensity of light is cut down to half due to polarisation, irrespective of the orientation of the polaroid. When polaroid is rotated, the intensity of light does not show any change.

54. Refer to the solution of Problem 40 on page 10.69.

(*i*) I_1 will remain unaffected whereas I_2 will decrease from maximum $(I_0/2)$ to zero of the incident light.

(*ii*) $I_2 = I_1 \cos^2\theta = \frac{I_0}{2}.\cos^2\theta.$

55. Refer answer to Q.25 on page 10.32.

TYPE C : LONG ANSWER QUESTIONS (5 marks each)

1. (*a*) State Huygen's principle. Using this principle draw a diagram to show how a plane wavefront incident at the interface of the two media gets refracted when it propagates from a rarer to a denser medium. Hence verify Snell's law of refraction.

(*b*) When monochromatic light travels from a rarer to a denser medium, explain the following, giving reasons :

(*i*) Is the frequency of reflected and refracted light same as the frequency of incident light ?

(*ii*) Does the decrease in speed imply a reduction in the energy carried by light wave ? [CBSE OD 05 ; D 13]

2. (*a*) Use Huygen's geometrical construction to show how a plane wavefront at $t = 0$ propagates and produces a wavefront at a later time.

(*b*) Verify, using Huygen's principle, Snell's law of refraction of a plane wave propagating from a denser to a rarer medium.

(*c*) Illustrate with the help of diagrams the action of (*i*) convex lens and (*ii*) concave mirror, on a plane wavefront incident on it. [CBSE D 13C]

3. State the importance of coherent sources in the phenomenon of interference.

In Young's double slit experiment to produce interference pattern, obtain the conditions for constructive and destructive interference. Hence deduce the expression for the fringe width. How does the fringe width get affected, if the entire experimental apparatus of Young is immersed in water ? [CBSE OD 11, 12]

4. (*a*) What are coherent sources of light ? Two slits of Young's double slit experiment are illuminated by two different sodium lamps emitting light of the same wavelength. Why is no interference pattern observed ?

(*b*) Obtain the condition for getting dark and bright fringes in Young's experiment. Hence write the expression for the fringe width. If the setup were to be put up in a medium optically denser than air, what effect would be there on the observed fringe width ? Give reason for your answer.

(*c*) If s is the size of the source and d its distance from the plane of the two slits, what should be the criterion for the interference fringes to be seen ? [CBSE OD 06, 07C, 08]

5. (a) In Young's double slit experiment, deduce the conditions for obtaining constructive and destructive interference fringes. Hence deduce the expression for the fringe width.
(b) Explain by drawing a suitable diagram that the interference pattern in a double slit is actually a superposition of single slit diffraction from each slit.
(c) What should be the width of each slit to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern, for green light of wavelength 500 nm, if the separation between two slits is 1 mm ? [CBSE OD 15]

6. In a Young's double slit experiment,
(a) deduce the conditions for constructive and destructive interference. Hence write the expression for the distance between two consecutive bright or dark fringes.
(b) what change in the interference pattern do you observe if the two slits, S_1 and S_2 are taken as point sources ?
(c) plot a graph of the intensity distribution vs. path difference in this experiment. Compare this with the intensity distribution of fringes due to diffraction at a single slit. What important difference do you observe ? [CBSE D 09C]

7. (a) What is the effect on the interference fringe in a Young's double slit experiment when (i) the separation between the two slits is decreased ? (ii) the width of a source slit is increased ? (iii) the monochromatic source is replaced by a source of white light ?
Justify your answer in each case.
(b) The intensity at the central maxima in Young's double slit experimental set-up is I_0. Show that the intensity at a point where the path difference is $\lambda / 3$ is $I_0 / 4$. [CBSE F 12]

8. A parallel beam of monochromatic light falls normally on a narrow slit and the light, coming out of the slit, is obtained on a screen, kept behind, parallel to the slit plane.
What kind of pattern do we observe on the screen and why ? How does the (i) angular width (ii) linear width of the principal maximum, in this pattern change when the screen is moved, parallel to itself, away from the slit plane ?
State two points of difference between this pattern and the interference pattern observed in the Young's double slit experiment. [CBSE D 08C]

9. (a) Using Huygens' construction of secondary wavelets explain how a diffraction pattern is obtained on a screen due to a narrow slit on which a monochromatic beam of light is incident normally.
(b) Show that the angular width of the first diffraction fringe is half that of the central fringe.
(c) Explain why the maxima at $\theta = \left(n + \frac{1}{2}\right)\frac{\lambda}{a}$ become weaker and weaker with increasing n. [CBSE D 15]

10. (a) Draw a ray diagram showing image formation in a compound microscope. Define the term 'limit of resolution' and name the factors on which it depends. How is it related to resolving power of a microscope ?
(b) Suggest two ways by which the resolving power of a microscope can be increased.
(c) "A telescope resolves whereas a microscope magnifies." Justify this statement. [CBSE F 15]

11. (a) Describe briefly how a diffraction pattern is obtained on a screen due to a single narrow slit illuminated by a monochromatic source of light. Hence obtain the conditions for the angular width of secondary maxima and secondary minima. [CBSE OD 14]
(b) 'Diffraction defines the limit of the ray optics'. Give a brief explanation of this statement. [CBSE Sample Paper 11]

12. (a) How does one demonstrate, using a suitable diagram, that unpolarised light when passed through a polaroid gets polarised ?
(b) A beam of unpolarised light is incident on a glass-air interface. Show, using a suitable ray diagram, that light reflected from the interface is totally polarised, when $\mu = \tan i_B$, where μ is the refractive index of glass with respect to air and i_B is the Brewster's angle. [CBSE D 14]

13. (a) Distinguish between linearly polarised and unpolarised light.
(b) Show that the light waves are transverse in nature.
(c) Why does light from a clear blue portion of the sky show a rise and fall of intensity when viewed through a polaroid which is rotated ? Explain by drawing the necessary diagram. [CBSE D 14C]

14. (a) What are polaroids ? How are they used to demonstrate that
(i) light waves are transverse in nature,
(ii) if an unpolarized light wave is incident, then light wave will get linearly polarized ?

(b) What is Brewster's angle ? When an unpolarized light is incident on a plane glass surface, what should be the angle of incidence so that the reflected and refracted rays are perpendicular to each other ? [CBSE D 08C]

15. (a) Define a 'linearly polarized' or 'plane polarized' light. Why is the phenomenon of polarization not observed by sound waves ?

(b) What does a polaroid consist of ? How does an unpolarized ligth incident on a polaroid get linearly polarized ?

(c) A beam of unpolarized light is made to fall, from air, on its boundary with another transparent medium of refractive index μ. The reflected light is viewed through a rotating analyzer.

Show, on a graph, the variation of intensity of light transmitted through the analyzer with the angle between the pass axis of the analyzer and the direction of the reflected beam. [CBSE D 09C]

16. (a) How does an unpolarized light incident on a polaroid get polarized ?

Describe briefly, with the help of a necessary diagram, the polarization of light by reflection from a transparent medium.

(b) Two polaroids 'A' and 'B' are kept in crossed position. How should a third polaroid 'C' be placed between them so that the intensity of polarized light transmitted by polaroid B reduces to 1/8th of the intensity of unpolarized light incident on A ? [CBSE OD 12]

Answers

1. (a) Refer answer to Q. 5 on page 10.4.

(b) (i) Refer to the solution of Problem 1(a) on page 10.61.

(ii) Refer to the solution of Problem 1(b) on page 10.61.

2. (a) Refer answe. to Q. 3 on page 10.3.

(b) Refer to the solution of Problem 4(a) on page 10.62.

(c) See Fig. 10.8 and Fig. 10.9 on page 10.5.

3. If the two sources are not coherent, then the phase difference between them will change 10^8 times per second. Such rapid changes in the positions of maxima and minima cannot be detected by our eyes. The interference pattern is lost and a uniform illumination is seen on the screen.

For expression of fringe width in Young's double slit experiment, refer answer to Q. 13 on page 10.13.

The wavelength of light in water ($\lambda' = \lambda/\mu$) is less than that in air. When the apparatus is immersed in water, fringe width ($\beta \propto \lambda'$) decreases.

4. (a) Refer to solution of Problem 15 on page 10.56.

(b) Refer answer to Q. 13 on page 10.13. Wavelength of light in optically denser medium decreases, hence fringe width decreases.

(c) For interference fringes to be seen, the condition $\frac{s}{d} < \frac{\lambda}{a}$ must be satisfied. Here 'a' is the distance between the two coherent sources.

5. (a) Refer answer to Q. 13 on page 10.13.

(b) Refer answer to Q. 27 on page 10.38.

(c) Clearly, 10β = width of central maximum

or $$10\frac{D\lambda}{d} = 2\frac{D\lambda}{a}$$

or $$a = \frac{d}{5} = \frac{1}{5}\text{mm} = \mathbf{0.2\ mm}.$$

6. (a) Refer answer to Q.10 on page 10.10.

(b) The straight line fringes will change into circular fringes.

(c) Refer to solution of Problem 19 on page 10.65.

7. (a) Refer to the solution of Problem 7 on page 10.63.

(b) If the intensity at the central maxima is I_0, then

$$I = I_0 \cos^2 \frac{\phi}{2}$$

For $p = \frac{\lambda}{3}$, $\phi = \frac{2\pi}{3}$,

so $$I = I_0 \cos^2 \frac{\pi}{3} = I_0\left(\frac{1}{2}\right)^2 = \frac{\mathbf{I_0}}{\mathbf{4}}.$$

8. The light on passing through the narrow slit undergoes diffraction. A diffraction pattern consisting of alternate bright and dark bands is obtained on the screen.

(i) Angular width of principal maximum,

$$2\theta = \frac{2\lambda}{a}.$$

It is not affected when screen is moved away (D increases) from the slit plane.

(ii) Linear width of principal maximum,

$$\beta_0 = \frac{2D\lambda}{a}.$$

It increases when the distance D increases. For differences, refer to the solution of Problem 19 on page 10.65.

9. (*a*) For diffraction of light at a single slit, refer answer to Q. 24 on page 10.30.

(*b*) The condition for first minimum on the screen is

$$a \sin\theta = \lambda \qquad \therefore\ \theta \simeq \sin\theta = \frac{\lambda}{a}$$

The angular width of the central fringe on the screen $= 2\theta = \dfrac{2\lambda}{a}$

Angular width of the first secondary diffraction fringe $= \dfrac{\lambda}{a}$

Hence the angular width of central fringe is twice the angular width of first secondary fringe.

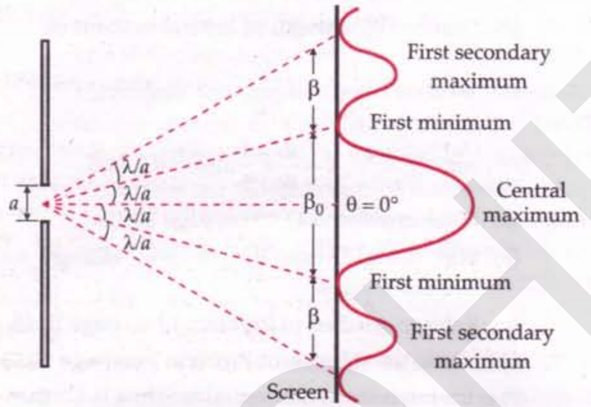


Fig. 10.81

(*c*) Maxima become weaker and weaker with increasing *n*. This is because the effective part of the wavefront, contributing to the maxima, becomes smaller and smaller, with the increasing *n*.

10. (*a*) See Fig. 9.145 on page 9.91 for image formation in a compound microscope.

Limit of resolution. The smallest linear or angular separation between two point objects at which they can be just separately seen or resolved by an optical instrument is called the limit of resolution of the instrument.

Factors on which limit of resolution depends :

(*i*) Wavelength of light used

(*ii*) Medium between object and objective.

Resolving power of microscope is equal to the reciprocal of the limit of resolution.

(*b*) Resolving power of a microscope can be increased by

(*i*) using light of smaller wavelength

(*ii*) using medium of high refractive index between the object and the objective of the microscope.

(*c*) A telescope produces angular magnification of the distant objects and thereby enables us to resolve them.

A microscope produces linear magnification and thereby enables us to view nearby tiny objects clearly.

11. (*a*) Refer answer to Q. 25 on page 10.32.

(*b*) When size of aperture $a >> \lambda$, we can ignore diffraction effects. Then the light appears to travel in straight lines and ray optics becomes valid. Since for the validity of ray optics $a >> \lambda$ or $\lambda << a$ or $\lambda \to 0$, it implies that ray optics is a limiting case of wave optics.

12. (*a*) Refer answer to Q. 37 on page 10.44.

(*b*) Refer answer to Q. 42 on page 10.48.

13. (*a*) Refer to points 30 and 31 of Glimpses on page 10.105.

(*b*) Refer answer to Q. 37 on page 10.44.

(*c*) See Fig. 10.46 on page 10.51.

It is due to the scattering of molecules of earth's atmosphere. Under the influence of the electric field of the incident (unpolarised) wave, the electrons in the molecules acquire components of motion in both of these directions. Charges, accelerating parallel to the double arrows, do not radiate energy towards the observer since their acceleration has no transverse component. The radiation scattered by the molecules is, therefore, represented by dots, *i.e.*, it is polarised perpendicular to plane of the figure.

14. (*a*) Refer answer to Q. 47 on page 10.52.

(*b*) Refer to the solution of Example 79 on page 10.49.

15. (*a*) Refer answer to Q.14 above. As sound waves are longitudinal, so they cannot be polarized.

(*b*) A polaroid consists of a thin stretched film of polymer like polyvinyl alcohol (PVA) whose molecules are dichroic. When unpolarised light is incident on a polaroid, it transmits only those vibrations which are parallel to axis and absorbs vibrations in the perpendicular direction. Thus the transmitted light is plane-polarized.

(*c*) See Fig. 10.39 on page 10.45.

16. (a) When unpolarised light falls on a polaroid, only the vibrations parallel to the transmission plane get transmitted and perpendicular vibrations are selectively absorbed. So the emergent light is plane polarised.

For polarisation by reflection, refer answer to Q. 42 on page 10.48.

(b) If the polarisation axis of C makes angle θ with the polarisation axis of A, then

$$I_C = \frac{1}{4} I_0 \sin^2 2\theta$$

But $I_C = \frac{I_0}{8}$

$$\therefore \quad \frac{I_0}{8} = \frac{1}{4} I_0 \sin^2 2\theta$$

or $\sin^2 2\theta = \frac{1}{2}$ or $\sin 2\theta = \frac{1}{\sqrt{2}}$

$$\therefore \quad 2\theta = 45^\circ$$

$$\Rightarrow \quad \theta = 22\tfrac{1}{2}^\circ$$

TYPE D : VALUE BASED QUESTIONS (4 marks each)

1. Geeta was watching her favourite programme KBC on TV. Suddenly the picture started shaking on the TV screen. She asked her elder brother to check the dish antenna. Her brother found nothing wrong with the dish. A little later, Geeta again noticed the same problem on the TV screen. At the same time, she heard the sound of a low flying aircraft passing over their house. She asked her brother again. He then explained her the cause of shaking picture on TV screen when aircraft passes overhead.
   (a) What are the values shown by Geeta's brother ?
   (b) Why do we observe slight shaking of the picture on our TV screen when a low-lying aircraft passes overhead ?
2. Rohan observed that thin films such as a soap bubble or a thin layer of oil on water show beautiful colours when illuminated by white light. He felt happy and surprised to see that. He went to his Physics teacher to understand the reason behind it. The teacher explained him that a thin film of oil spread over water shows interference of light due to the interference between the light waves reflected by the lower and upper surfaces of the thin film. On understanding this phenomenon well, Rohan then gave an example of thin film of kerosene oil which is spread over water to prevent malaria and dengue.
   (a) What are the values being displayed by Rohan's teacher ?
   (b) Why does a soap bubble show beautiful colours ?
3. Jivin observed that when a sheet of transparent plastic is placed between two crossed polarizers, no light is transmitted. When the sheet is stretched in one direction, some light passes through the crossed polarizers. He was surprised to see that and out of shear enthusiasm, he went to his Physics teacher for knowing the reason behind it. The teacher explained him that the stretched plastic sheet turns into a polaroid and allows a fraction of light pass through it.
   (a) What are the values being displayed by Jivin here ?
   (b) When the plastic sheet is stretched in one direction, some light passes through the crossed polarizers. What is happening ?

Answer

1. (a) Critical thinking and problem solving.
   (b) The low lying aircraft reflects the TV signal. Due to interference between direct signal received by antenna and the weak reflected signal, slight shaking of the picture is seen on the screen.
2. (a) Appreciation of nature and motivation.
   (b) Light waves reflected from the upper and lower surfaces of thin film interfere. Since the conditions of bright and dark fringes are satisfied at different positions for different wavelengths, so coloured fringes are seen.
3. (a) Power of observation and curiosity.
   (b) When the sheet is stretched, the polymer molecules in it make it polaroid with its own polaroid axis. Now we have three polaroids with the middle polaroid having its axis between the axes of the two outer polaroids. That is why some light is transmitted in this case.

COMPETITION SECTION

Wave Optics

GLIMPSES

1. Nature of light. The phenomena like interference, diffraction and polarisation establish the wave nature of light. However, the phenomena like black body radiation and photoelectric effect establish the particle nature of light. de Broglie suggested that *light has a dual nature i.e.,* it can behave as particles as well as waves.

2. Wavefront. A wavefront is defined as the continuous locus of all such particles of the medium which are vibrating in the same phase at any instant. In case of waves travelling in all directions from a point source, the wavefronts are spherical in shape. When the source of light is linear in shape, the wavefronts are cylindrical. At very large distances from the source, a portion of spherical or cylindrical wavefront is plane wavefront.

3. Ray. An arrow drawn perpendicular to a wavefront in the direction of propagation of a wave is called a ray.

   Two general principles are valid for rays and wavefronts :

   (*i*) Rays are normal to wavefronts.

   (*ii*) The time taken to travel from one wavefront to another is the same along any ray.

4. Huygens' principle of secondary wavelets. Huygens' principle is the basis of the wave theory of light. It tells how a wavefront propagates through a medium. It is based on the following assumptions :

   (*i*) Each point on a wavefront acts as a source of new disturbance called secondary waves or wavelets.

   (*ii*) The secondary wavelets spread out in all directions with the speed of light in the given medium.

   (*iii*) The wavefront at any later time is given by the forward envelope of the secondary wavelets at that time.

5. Effect on frequency, wavelength and speed during refraction. When a light wave travels from one medium to another, its frequency remains unchanged but both its wavelength and speed get changed, depending on the refractive index of the refracting medium.

6. Interference of light waves. When light waves from two coherent sources travelling in the same direction superpose each other, the intensity in the region of superposition gets redistributed, becoming maximum at some points and minimum at others. This phenomenon is called interference of light.

7. Constructive and destructive interference. If path difference $p = n\lambda$ or phase difference $\phi = 2n\pi$, the two waves are in same phase and so add up to give maximum of intensity. This is called *constructive interference.*

   If $p = (2n-1)\lambda/2$ or $\phi = (2n-1)\pi$, the two superposing waves are out of phase, the resultant amplitude is equal to difference between their individual amplitudes and hence intensity is minimum. This is called *destructive interference.*

8. Young's double slit experiment. In Young's double slit experiment, two identical narrow slits S_1 and S_2 are placed symmetrically with respect to narrow slit S illuminated with

monochromatic light. The interference pattern is obtained on an observation screen placed at large distance D from S_1 and S_2.

The position of nth bright fringe from the centre of screen is

$$x_n = n\frac{D\lambda}{d}$$

The position of nth dark fringe from the centre of the screen is

$$x'_n = (2n-1)\frac{D\lambda}{2d}$$

Fringe width is the separation between two successive bright or dark fringes and is given by

$$\beta = \frac{D\lambda}{d}$$

9. Resultant amplitude and intensity of interfering waves. If a_1 and a_2 are the amplitudes and I_1 and I_2 are the intensities of two coherent waves having phase difference ϕ, then their resultant amplitude and intensity at the point of superposition are given by

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$$

and $\quad I = I_1 + I_2 + 2\cdot\sqrt{I_1 I_2}\cos\phi$

If amplitude of each wave is a_0 and intensity I_0, then

$$I = 2ka_0^2(1+\cos\phi) = 2I_0(1+\cos\phi)$$
$$= 4I_0\cos^2\frac{\phi}{2}$$

The term $2\sqrt{I_1 I_2}\cos\phi$ is called *interference term*.

(*i*) When $\cos\phi$ remains constant with time, the two sources are coherent. The intensity will be maximum at points for which $\cos\phi = +1$ and minimum at points for which $\cos\phi = -1$.

(*ii*) When $\cos\phi$ varies continuously with time so that its average value is zero over the time interval of measurement, the resultant intensity at all points will be $I_1 + I_2$. No interference fringes are observed. The sources are incoherent.

10. Ratio of intensity at maxima and minima of an interference pattern. If a_1 and a_2 are the amplitudes of two interfering waves, then the ratio between the intensities at maxima and minima will be

$$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \left(\frac{r+1}{r-1}\right)^2$$

where $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}}$, is the amplitude ratio of two waves. If w_1 and w_2 are the widths of the two slits, then

$$\frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}.$$

11. Coherent sources. Two sources of light which continuously emit light waves of same frequency (or wavelength) with a zero or constant phase difference between them, are called coherent sources. Two independent sources of light cannot act as coherent sources, they have to be derived from the same parent source.

12. Conditions for substained interference :

(*i*) The two sources should continuously emit waves of same frequency or wavelength.

(*ii*) The two sources of light should be coherent.

(*iii*) The amplitudes of the interfering waves should be equal.

(*iv*) The two sources should be narrow.

(*v*) The interfering waves must travel nearly along the same direction.

(*vi*) The sources should be monochromatic.

(*vii*) The interfering waves should be in the same state of polarisation.

(*viii*) The distance between the two coherent sources should be small and the distance between the two sources and the screen should be large.

13. Fresnel's biprism method. Here two coherent sources are obtained from an incoherent source, by refraction. A biprism is essentially a single prism with an obtuse angle of 179°, but behaves as a combination of two acute angled prisms placed base to base, each with a refracting angle of about $\frac{1°}{2}$.

14. Lloyd's single mirror method. In this method, an illuminated slit and its reflected image serve as two coherent sources. In contrast to Young's double slit and Fresnel's biprism methods, here the central fringe is dark.

15. Displacement of interference fringes. When a thin transparent sheet of thickness t and refractive index μ is inserted in the path of one of the interfering beams, the extra path difference introduced is

$$\Delta p = \text{Length } t \text{ in transparent sheet} - \text{Length } t \text{ in air}$$

or $\Delta p = \mu t - t = (\mu - 1)t$

$\therefore$ Net path difference for any point on the screen

$$= \frac{xd}{D} - (\mu - 1)t$$

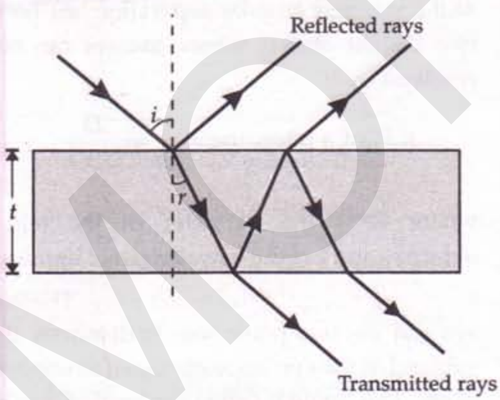
For the central point of the screen,

$$\frac{xd}{D} - (\mu - 1)t = 0 \quad \text{or} \quad x = \frac{D}{d}(\mu - 1)t$$

Thus the shift in the central bright fringe and hence shift of any other fringe is

$$\Delta x = \frac{D}{d}(\mu - 1)t = \frac{\beta}{\lambda}(\mu - 1)t$$

16. Interference in thin films. A soap film or thin film of oil spread over water shows beautiful colours, when seen in the reflected light. This is due to interference between light waves reflected by the upper and lower surfaces of thin films, as shown in the figure below. The ray reflected from the upper denser surface of thin film suffers a phase change of π or path difference of $\lambda/2$.



Reflected system. The path difference between the two consecutive rays reflected from the upper and the lower surfaces of a thin film of refractive index μ and thickness t is given by

$$p = 2\mu t \cos r - \frac{\lambda}{2}$$

For maximum intensity. $2\,\mu t \cos r = (2n + 1)\frac{\lambda}{2}$

For minimum intensity. $2\,\mu t \cos r = n\lambda$.

Transmitted system.

For maximum intensity. $2\mu t \cos r = n\lambda$

For minimum intensity.

$$2\,\mu t \cos r = (2n + 1)\frac{\lambda}{2}, \text{ where } n = 0, 1, 2, 3....$$

17. Diffraction of light. The phenomenon of bending of light around the corners of small obstacles or apertures and their consequent spreading into the regions of geometrical shadow is called diffraction of light.

18. Diffraction at a single slit. A plane wave of wavelength λ on passing through a narrow slit of width a suffers diffraction producing a central bright fringe ($\theta = 0°$), flanked on both sides by minima and maxima. The intensity of secondary maxima decreases with the increase in distance from the centre.

For nth minimum :

$$a \sin \theta_n = n\lambda, \qquad n = 1, 2, 3, ...$$

For nth secondary maximum :

$$a \sin \theta_n = (2n + 1)\frac{\lambda}{2}, \quad n = 1, 2, 3, ...$$

Angular position of nth minimum,

$$\theta_n = \frac{n\lambda}{a}$$

Distance of nth minimum from the centre of the screen,

$$x_n = n\frac{D\lambda}{a}$$

Angular position of nth secondary maximum,

$$\theta'_n = (2n + 1)\frac{\lambda}{2a}$$

Distance of nth secondary maximum from the centre of the screen,

$$x'_n = (2n + 1)\frac{D\lambda}{2a}$$

Width of a secondary maximum,

$$\beta = \frac{D\lambda}{a}$$

Width of central maximum,

$$\beta_0 = 2\beta = \frac{2D\lambda}{a}$$

Angular spread of central maximum on either side of the centre of the screen is

$$\theta = \pm\frac{\lambda}{a}$$

Total angular spread of the central maximum is

$$2\,\theta = \frac{2\,\lambda}{a}$$

For diffraction to be more pronounced, the size of the slit should be comparable to the wavelength of light used.

19. Diffraction at a circular aperture. For diffraction of light at a circular aperture of diameter *a*, the angular spread of central maximum is

$$\theta = \frac{1.22\,\lambda}{a}$$

If *D* is the distance at which the effect is observed, then

Linear spread, $x = D\theta$

Areal spread, $x^2 = (D\theta)^2$.

20. Fresnel's distance. It is the distance at which the diffraction spread of a beam becomes equal to the size of the aperture. If *a* is the width of the aperture, then

$$D_F = \frac{a^2}{\lambda}$$

The ray optics is valid for a distance $D < D_F$.

21. Diffraction grating. It is an arrangement of a very large number of very narrow, equidistant and parallel slits. The diffraction pattern has the central principal maximum of maximum intensity and a number of higher order intensity maxima whose intensity decreases with the increase of *n*, the order of the spectrum. The direction of *n*th principal maximum is given by

$$(a + b)\sin\theta_n = n\lambda$$

where $n = 0, 1, 2, 3 ....$

This equation is known as *grating law*. Here $(a + b)$ is called *grating element*, where *a* is width of each slit and *b* is the width of opaque space between two consecutive slits.

22. Limit of resolution. The smallest linear or angular separation between two point objects at which they can be just resolved by an optical instrument is called the limit of resolution of the instrument.

23. Resolving power. It is the ability of an optical instrument to resolve or separate the images of two nearby point objects so that they can be distinctly seen. It is equal to the reciprocal of the limit of resolution of the optical instrument.

24. Diffraction as a limit on resolving power. All optical instruments like lens, telescope, microscope, etc. act as apertures. Light on passing through them undergoes diffraction. This puts the limit on their resolving power.

25. Rayleigh's criterion for resolution. The images of two point objects are just resolved when the central maximum of the diffraction pattern of one falls over the first minimum of the diffraction pattern of the other.

26. Resolving power of a microscope. The resolving power of a microscope is defined as the reciprocal of the smallest distance *d* between two point objects at which they can be just resolved when seen in the microscope.

$$\text{R.P. of a microscope} = \frac{1}{d} = \frac{2\mu\sin\theta}{\lambda}$$

where θ is half the angle of cone of light from each point object and μ is the refractive index of the medium between the object and the objective. The factor μ sin θ is called *numerical aperture* (N.A.).

27. Resolving power of a telescope. The resolving power of a telescope is defined as the reciprocal of the smallest angular separation '*d*θ' between two distant objects whose images can be just resolved by it.

$$\text{R.P. of a telescope} = \frac{1}{d\theta} = \frac{D}{122\,\lambda}$$

where *D* is the diameter of the telescope objective and λ is the wavelength of light used.

28. Resolving power of the human eye. The human eye can see two point objects distinctly if they subtend at the eye, an angle equal to one minute of arc. This angle is called the limit of resolution of the eye. The reciprocal of this angle equals the resolving power of the eye.

29. **Polarisation of waves.** A transverse wave in which vibrations are present in all possible directions, in a plane perpendicular to the direction of propagation, is said to be unpolarised. If the vibrations of a wave are present in just one direction in a plane perpendicular to the direction of propagation, the wave is said to be polarised or plane polarised. The phenomenon of restricting the oscillations of a wave to just one direction in the transverse plane is called polarisation.

30. Unpolarised light. A kind of light in which the electric field vector takes all possible directions in the transverse plane, rapidly and randomly, during the time of measurement is called unpolarised light. For example, the light of the sun, candle light, etc.

31. Plane polarised light. If the electric field vector vibrates just in one direction perpendicular to the direction of wave propagation, the light is said to be linearly polarised. In a linearly polarised wave, the vibrations at all points, at all times, lie in the same plane, so it is also called a plane polarised wave.

32. Polariser. A device that plane polarises the unpolarised light passed through it is called a polariser. For example, a tourmaline crystal, nicol prism, polaroid, etc.

33. Law of Malus. This law states that when a beam of completely plane polarised light is passed through an analyser, the intensity 'I' of the transmitted light varies directly as the square of the angle 'θ' between the transmission directions of polariser and analyser.

$$I = I_0 \cos^2 \theta$$

where I_0 is the maximum intensity of transmitted light.

34. Plane of polarisation. The plane passing through the direction of wave propagation and perpendicular to the plane of vibration is called the plane of polarisation.

35. Plane of vibration. The plane containing the direction of vibration and the direction of wave propagation is called the plane of vibration.

36. Brewster angle. The angle of incidence at which a beam of unpolarised light falling on a transparent surface is reflected as a beam of completely plane polarised light is called polarising or Brewster angle. It is denoted by i_p.

37. Brewster law. This law states that the tangent of the polarising angle of incidence of a transparent medium is equal to its refractive index.

$$\mu = \tan i_p$$

38. Nicol prism. It is an optical device based on the phenomenon of double refraction which is used for producing and analysing plane polarised light. It consists of two pieces of calcite cut with a 68° angle and stuck together with Canada balsam.

39. Polaroids. These are thin commercial sheets which make use of the property of selective absorption (dichroism) to produce an intense beam of plane polarised light. Polaroids are used in sunglasses, camera filters, wind screens and car head lights of motor cars to reduce glare of light reflected from shiny surfaces, etc.

40. Optical activity. Substances which can rotate the plane of polarisation of light are called optically active substances while the phenomenon is called optical activity.

41. Specific rotation. It is the angle through which the plane of polarisation rotates when plane polarised light is passed through one decimetre length of solution containing one gram of the substance per cm^3. The measurement is done at a given temperature T, using sodium light (the D-line).

Specific rotation

$$= \frac{\text{Observed angle of rotation in degrees}}{\text{Length of the tube in decimetre} \times \text{Grams of substance in 1 cm}^3 \text{ of solution}}$$

$$[s]_D^T = \frac{\theta}{l \times c}$$

42. Doppler effect. It is the phenomenon of the apparent change in the frequency of light due to the relative motion between the source and observer. The apparent frequency ν' is given by

$$\nu' = \nu\left(1 \pm \frac{v}{c}\right)$$

When source moves towards the observer, velocity v is taken $+ve$ and when it moves away from the observer, v is taken $-ve$.

43. Doppler shift. The apparent change in the frequency of light due to Doppler effect is called Doppler shift.

(*i*) $\Delta \nu = \pm \frac{v}{c} \nu$

(*ii*) $\Delta \lambda = \mp \frac{v}{c} . \lambda .$